

Chapter 07.02

Trapezoidal Rule for Integration-More Examples

Mechanical Engineering

Example 1

A trunnion of diameter 12.363" has to be cooled from a room temperature of 80°F before it is shrink fit into a steel hub (Figure 1).

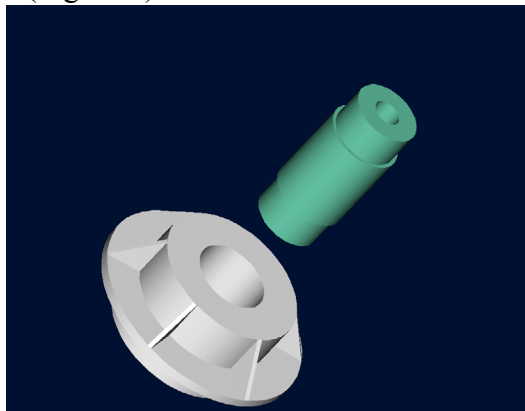


Figure 1 Trunnion to be slid through the hub after contracting.

The equation that gives the diametric contraction, in inches of the trunnion in dry-ice/alcohol (boiling temperature is -108°F) is given by:

$$\Delta D = 12.363 \int_{80}^{-108} (-1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.015 \times 10^{-6}) dT$$

- Use single segment Trapezoidal rule to find the contraction.
- Find the true error, E_t , for part (a).
- Find the absolute relative true error for part (a).

Solution

$$a) \quad I \approx (b - a) \left[\frac{f(a) + f(b)}{2} \right], \text{ where}$$

$$a = 80$$

$$b = -108$$

$$f(T) = 12.363(-1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.015 \times 10^{-6})$$

$$f(80) = 12.363(-1.2278 \times 10^{-11} (80)^2 + 6.1946 \times 10^{-9} (80) + 6.015 \times 10^{-6})$$

$$= 7.9519 \times 10^{-5}$$

$$f(-108) = 12.363(-1.2278 \times 10^{-11} (-108)^2 + 6.1946 \times 10^{-9} (-108) + 6.015 \times 10^{-6})$$

$$\begin{aligned}
 &= 6.4322 \times 10^{-5} \\
 I &\approx (-108 - 80) \left[\frac{7.9519 \times 10^{-5} + 6.4322 \times 10^{-5}}{2} \right] \\
 &\approx -0.013521 \text{ in}
 \end{aligned}$$

b) The exact value of the above integral is

$$\begin{aligned}
 \Delta D &= 12.363 \int_{80}^{-108} (-1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.015 \times 10^{-6}) dT \\
 &= -0.013689 \text{ in}
 \end{aligned}$$

so the true error is

$$\begin{aligned}
 E_t &= \text{True Value} - \text{Approximate Value} \\
 &= -0.013689 - (-0.013521) \\
 &= -0.00016810
 \end{aligned}$$

c) The absolute relative true error, $|\epsilon_t|$, would then be

$$\begin{aligned}
 |\epsilon_t| &= \left| \frac{\text{True Error}}{\text{True Value}} \right| \times 100 \% \\
 &= \left| \frac{-0.00016810}{-0.013689} \right| \times 100 \% \\
 &= 1.2280 \%
 \end{aligned}$$

Example 2

A trunnion of diameter 12.363" has to be cooled from a room temperature of 80°F before it is shrink fit into a steel hub (Figure 1). The equation that gives the diametric contraction, in inches of the trunnion in dry-ice/alcohol (boiling temperature is -108°F) is given by:

$$\Delta D = 12.363 \int_{80}^{-108} (-1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.015 \times 10^{-6}) dT$$

Use two segment Trapezoidal rule to find the contraction.

- Find the true error, E_t , for part (a).
- Find the absolute relative true error for part (a).

Solution

a) The solution using 2-segment Trapezoidal rule is

$$\begin{aligned}
 I &\approx \frac{b-a}{2n} \left[f(a) + 2 \left\{ \sum_{i=1}^{n-1} f(a+ih) \right\} + f(b) \right] \\
 n &= 2 \\
 a &= 80 \\
 b &= -108 \\
 h &= \frac{b-a}{n} \\
 &= \frac{-108-80}{2}
 \end{aligned}$$

$$\begin{aligned}
&= -94 \\
I &\approx \frac{-108 - 80}{2(2)} \left[f(80) + 2 \left\{ \sum_{i=1}^{2-1} f(a + ih) \right\} + f(-108) \right] \\
&\approx \frac{-188}{4} [f(80) + 2f(80 + 1 \times (-94)) + f(-108)] \\
&\approx \frac{-188}{4} [f(80) + 2f(-14) + f(-108)] \\
&\approx -47 [7.9519 \times 10^{-5} + 2(7.3262 \times 10^{-5}) + 6.4322 \times 10^{-5}] \\
&\approx -0.013647 \text{ in}
\end{aligned}$$

b) The exact value of the above integral is

$$\begin{aligned}
\Delta D &= 12.363 \int_{80}^{-108} (-1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.015 \times 10^{-6}) dT \\
&= -0.013689 \text{ in}
\end{aligned}$$

so the true error is

$$\begin{aligned}
E_t &= \text{True Value} - \text{Approximate Value} \\
&= -0.013689 - (-0.013647) \\
&= -0.000042026 \text{ in}
\end{aligned}$$

c) The absolute relative true error, $|\epsilon_t|$, would then be

$$\begin{aligned}
|\epsilon_t| &= \left| \frac{\text{True Error}}{\text{True Value}} \right| \times 100 \% \\
&= \left| \frac{-0.000042026}{-0.013689} \right| \times 100 \% \\
&= 0.30700 \%
\end{aligned}$$

Table 1 Values obtained using multiple-segment Trapezoidal rule for

$$\Delta D = 12.363 \int_{80}^{-108} (-1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.015 \times 10^{-6}) dT$$

n	Value	E_t	$ \epsilon_t \%$	$ \epsilon_a \%$
1	-0.013521	-0.00016810	1.2280	---
2	-0.013647	-4.2026×10^{-5}	0.30700	0.92328
3	-0.013670	-1.8678×10^{-5}	0.13644	0.16825
4	-0.013679	-1.0506×10^{-5}	0.076750	0.059740
5	-0.013682	-6.7241×10^{-6}	0.049120	0.027644
6	-0.013684	-4.6695×10^{-6}	0.034111	0.015014
7	-0.013686	-3.4307×10^{-6}	0.025061	0.0090522
8	-0.013687	-2.6266×10^{-6}	0.019188	0.0058749