

Chapter 06.03

Linear Regression-More Examples

Mechanical Engineering

Example 1

The coefficient of thermal expansion, α , of steel is given at discrete values of temperature in Table 1.

Table 1 Coefficient of thermal expansion versus temperature for steel.

Temperature, T °F	Coefficient of thermal expansion, α in/in°F
80	6.470×10^{-6}
60	6.360×10^{-6}
40	6.240×10^{-6}
20	6.120×10^{-6}
0	6.000×10^{-6}
-20	5.860×10^{-6}
-40	5.720×10^{-6}
-60	5.580×10^{-6}
-80	5.430×10^{-6}
-100	5.280×10^{-6}
-120	5.090×10^{-6}
-140	4.910×10^{-6}
-160	4.720×10^{-6}
-180	4.520×10^{-6}
-200	4.300×10^{-6}
-220	4.080×10^{-6}
-240	3.830×10^{-6}
-260	3.580×10^{-6}
-280	3.330×10^{-6}
-300	3.070×10^{-6}
-320	2.760×10^{-6}
-340	2.450×10^{-6}

The data is regressed to a first order polynomial.

$$\alpha = k_1 + k_2 T$$

Find the constants k_1 and k_2 of the regression model.

Solution

Table 2 shows the summations needed for the calculation of the constants of the regression model.

Table 2 Tabulation of data for calculation of needed summations.

I	T	α	$T\alpha$	T^2
–	°F	in/in/°F	in/in/°F	(°F) ²
1	80	6.470×10^{-6}	5.1760×10^{-4}	6400
2	60	6.360×10^{-6}	3.8160×10^{-4}	3600
3	40	6.240×10^{-6}	2.4960×10^{-4}	1600
4	20	6.120×10^{-6}	1.2240×10^{-4}	400
5	0	6.000×10^{-6}	0.000	0
6	-20	5.860×10^{-6}	-1.1720×10^{-4}	400
7	-40	5.720×10^{-6}	-2.2880×10^{-4}	1600
8	-60	5.580×10^{-6}	-3.3480×10^{-4}	3600
9	-80	5.430×10^{-6}	-4.3440×10^{-4}	6400
10	-100	5.280×10^{-6}	-5.2800×10^{-4}	10000
11	-120	5.090×10^{-6}	-6.1080×10^{-4}	14400
12	-140	4.910×10^{-6}	-6.8740×10^{-4}	19600
13	-160	4.720×10^{-6}	-7.5520×10^{-4}	25600
14	-180	4.520×10^{-6}	-8.1360×10^{-4}	32400
15	-200	4.300×10^{-6}	-8.6000×10^{-4}	40000
16	-220	4.080×10^{-6}	-8.9760×10^{-4}	48400
17	-240	3.830×10^{-6}	-9.1920×10^{-4}	57600
18	-260	3.580×10^{-6}	-9.3080×10^{-4}	67600
19	-280	3.330×10^{-6}	-9.3240×10^{-4}	78400
20	-300	3.070×10^{-6}	-9.2100×10^{-4}	90000
21	-320	2.760×10^{-6}	-8.8320×10^{-4}	102400
22	-340	2.450×10^{-6}	-8.3300×10^{-4}	115600
$\sum_{i=1}^{22}$	-2860	1.0570×10^{-4}	-1.0416×10^{-2}	726000

$$\begin{aligned}
 n &= 22 \\
 k_2 &= \frac{n \sum_{i=1}^{22} T_i \alpha_i - \sum_{i=1}^{22} T_i \sum_{i=1}^{22} \alpha_i}{n \sum_{i=1}^{22} T_i^2 - \left(\sum_{i=1}^{22} T_i \right)^2} \\
 &= \frac{22(-1.0416 \times 10^{-2}) - (-2860)(1.0570 \times 10^{-4})}{22(726000) - (-2860)^2} \\
 &= 9.3868 \times 10^{-9} \text{ in/in}/(^{\circ}\text{F})^2 \\
 \bar{\alpha} &= \frac{\sum_{i=1}^{22} \alpha_i}{n} \\
 &= \frac{1.0570 \times 10^{-4}}{22} \\
 &= 4.8045 \times 10^{-6} \text{ in/in}^{\circ}\text{F}
 \end{aligned}$$

$$\begin{aligned}
 \bar{T} &= \frac{\sum_{i=1}^{22} T_i}{n} \\
 &= \frac{-2860}{22} \\
 &= -130^{\circ}\text{F}
 \end{aligned}$$

$$\begin{aligned}
 k_1 &= \bar{\alpha} - k_2 \bar{T} \\
 &= 4.8045 \times 10^{-6} - (9.3868 \times 10^{-9})(-130) \\
 &= 6.0248 \times 10^{-6} \text{ in/in}^{\circ}\text{F}
 \end{aligned}$$

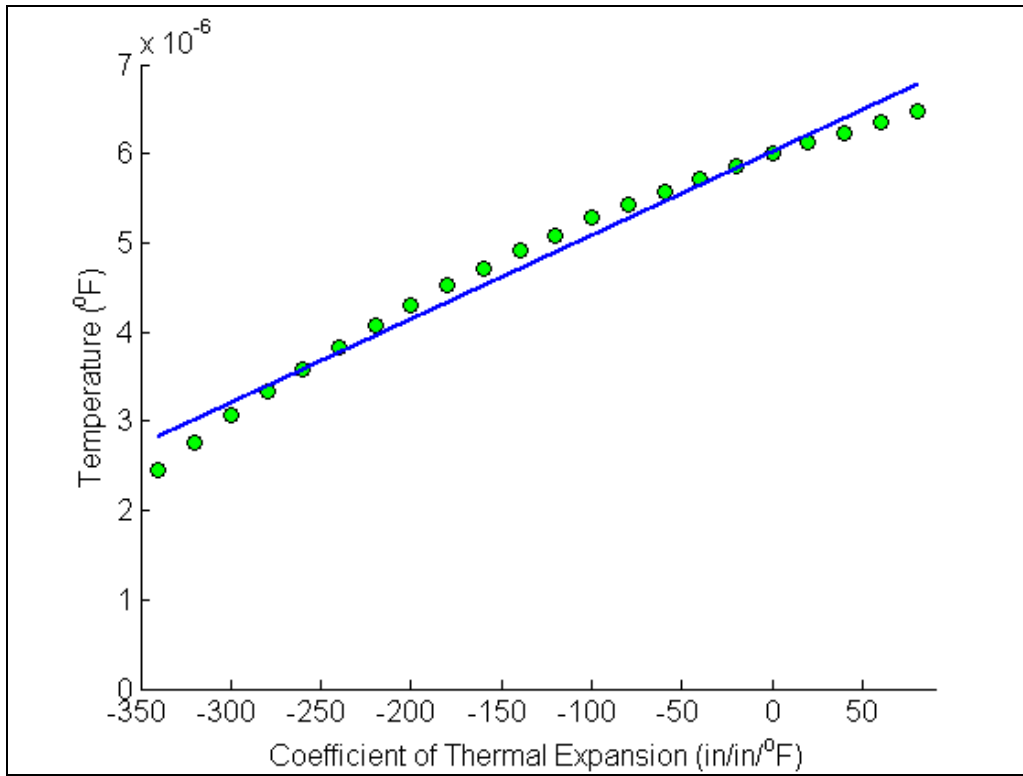


Figure 4 Linear regression of coefficient of thermal expansion vs. temperature data.