Chapter 07.02
Trapezoidal Rule for Integration-More Examples
Industrial Engineering

Example 1

A company advertises that every roll of toilet paper has at least 250 sheets. The probability that there are 250 or more sheets in the toilet paper is given by

$$P(y \geq 250) = \int_{250}^{\infty} 0.3515 \ e^{-0.3881(y-252.2)^2} dy$$

Approximating the above integral as

$$P(y \geq 250) = \int_{250}^{270} 0.3515 \ e^{-0.3881(y-252.2)^2} dy$$

a) Use single segment Trapezoidal rule to find the probability that there are 250 or more sheets.
b) Find the true error, E_t, for part (a).
c) Find the absolute relative true error for part (a).

Solution

a) $$I \approx (b-a)\left[\frac{f(a)+f(b)}{2}\right], \text{ where}$$

$$a = 250$$
$$b = 270$$
$$f(y) = 0.3515e^{-0.3881(y-252.2)^2}$$
$$f(250) = 0.3515e^{-0.3881(250-252.2)^2}$$
$$= 0.053721$$
$$f(270) = 0.3515e^{-0.3881(270-252.2)^2}$$
$$= 1.3888 \times 10^{-54}$$
$$I \approx (270-250)\left[\frac{0.053721 + 1.3888 \times 10^{-54}}{2}\right]$$
$$\approx 0.53721$$

b) The exact value of the above integral cannot be found. For calculating the true error and relative true error, we assume the value obtained by adaptive numerical integration using Maple as the exact value.

$$P(y \geq 250) = \int_{250}^{270} 0.3515 \ e^{-0.3881(y-252.2)^2} dy$$
$$= 0.97377$$

so the true error is

$$E_t = \text{True Value} - \text{Approximate Value}$$
$$= 0.97377 - 0.53721$$
$$= 0.43656$$

c) The absolute relative true error, $|\in_t|$, would then be

$$|\in_t| = \left|\frac{\text{True Error}}{\text{True Value}}\right| \times 100\,\%$$

$$= \left|\frac{0.97377 - 0.53721}{0.97377}\right| \times 100\,\%$$
$$= 44.832\,\%$$

Example 2

A company advertises that every roll of toilet paper has at least 250 sheets. The probability that there are 250 or more sheets in the toilet paper is given by

$$P(y \geq 250) = \int_{250}^{\infty} 0.3515\ e^{-0.3881(y-252.2)^2} dy$$

Approximating the above integral as

$$P(y \geq 250) = \int_{250}^{270} 0.3515\ e^{-0.3881(y-252.2)^2} dy$$

a) Use single segment Trapezoidal rule to find the probability that there are 250 or more sheets.
b) Find the true error, E_t for part (a).
c) Find the absolute relative true error for part (a).

Solution

a) $$I = \frac{b-a}{2n}\left[f(a) + 2\left\{\sum_{i=1}^{n-1} f(a+ih)\right\} + f(b)\right]$$

$$n = 2$$
$$a = 250$$
$$b = 270$$
$$h = \frac{b-a}{n}$$
$$= \frac{270-250}{2}$$
$$= 10$$

$$f(y) = 0.3515e^{-0.3881(y-252.2)^2}$$

$$I \approx \frac{270-250}{2(2)}\left[f(250) + 2\left\{\sum_{i=1}^{2-1} f(a+ih)\right\} + f(270)\right]$$

$$\approx \frac{20}{4}\left[f(250) + 2f(250 + 1\times 10) + f(270)\right]$$

$$\approx \frac{20}{4}\left[f(250)+2f(260)+f(270)\right]$$
$$\approx \frac{20}{4}\left[0.053721+2\left(1.9560\times10^{-11}\right)+1.3888\times10^{-54}\right]$$
$$\approx 0.26861$$

Since

$$f(250)=0.3515e^{-0.3881(250-252.2)^2}$$
$$=0.05372$$
$$f(270)=0.3515e^{-0.3881(270-252.2)^2}$$
$$=1.3888\times10^{-54}$$
$$f(260)=0.3515e^{-0.3881(260-252.2)^2}$$
$$=1.9560\times10^{-11}$$

b) The exact value of the above integral cannot be found. For calculating the true error and relative true error, we assume the value obtained by adaptive numerical integration using Maple as the exact value.

$$P(y\geq 250)=\int_{250}^{270}0.3515\ e^{-0.3881(y-252.2)^2}dy$$
$$=0.97377$$

so the true error is

$$E_t = \text{True Value} - \text{Approximate Value}$$
$$=0.97377-0.26861$$
$$=0.70516$$

c) The absolute relative true error, $|\in_t|$, would then be

$$|\in_t| = \left|\frac{\text{True Error}}{\text{True Value}}\right|\times100\,\%$$
$$=\left|\frac{0.97377-0.26861}{0.97377}\right|\times100\,\%$$
$$=72.416\,\%$$

Table 1 Values obtained using multiple-segment Trapezoidal rule for

$$P(y\geq 250)=\int_{250}^{270}0.3515\ e^{-0.3881(y-252.2)^2}dy$$

n	Value	E_t	$\lvert\in_t\rvert\,\%$	$\lvert\in_a\rvert\,\%$
1	0.53721	0.43656	44.832	---
2	0.26861	0.70516	72.416	99.999
3	0.18009	0.79368	81.506	49.153
4	0.21815	0.75562	77.598	17.447
5	0.50728	0.46648	47.905	56.997
6	0.80177	0.17200	17.663	36.729
7	0.93439	0.039381	4.0442	14.193
8	0.95768	0.016092	1.6525	2.4317