Chapter 08.02
Euler's Method for Ordinary Differential Equations-More Examples
Computer Engineering

Example 1

A rectifier-based power supply requires a capacitor to temporarily store power when the rectified waveform from the AC source drops below the target voltage. To properly size this capacitor a first-order ordinary differential equation must be solved. For a particular power supply, with a capacitor of $150\ \mu\text{F}$, the ordinary differential equation to be solved is

$$\frac{dv(t)}{dt} = \frac{1}{150\times 10^{-6}}\left\{-0.1 + \max\left(\frac{\left|18\cos(120\pi(t))\right| - 2 - v(t)}{0.04}, 0\right)\right\}$$
$$v(0) = 0$$

Using Euler's method, find the voltage across the capacitor at $t = 0.00004\,\text{s}$. Use step size $h = 0.00002\ \text{s}$.

Solution

$$\frac{dv}{dt} = \frac{1}{150\times 10^{-6}}\left\{-0.1 + \max\left(\frac{\left|18\cos(120\pi(t))\right| - 2 - v}{0.04}, 0\right)\right\}$$
$$f(t,v) = \frac{1}{150\times 10^{-6}}\left\{-0.1 + \max\left(\frac{\left|18\cos(120\pi(t))\right| - 2 - v}{0.04}, 0\right)\right\}$$

The Euler's method reduces to

$$v_{i+1} = v_i + f(t_i, v_i)h$$

For $i = 0$, $t_0 = 0$, $v_0 = 0$

$$\begin{aligned}
v_1 &= v_0 + f(t_0, v_0)h \\
&= 0 + f(0,0)0.00002 \\
&= \frac{1}{150\times 10^{-6}}\left\{-0.1 + \max\left(\frac{\left|18\cos(120\pi(0))\right| - 2 - (0)}{0.04}, 0\right)\right\}0.00002 \\
&= 0 + \left(2.666\times 10^{6}\right)0.00002 \\
&= 53.320\,\text{V}
\end{aligned}$$

v_1 is the approximate voltage at

$$t = t_1 = t_0 + h = 0 + 0.00002 = 0.00002 \text{ s}$$

$$v(0.00002) \approx v_1 = 53.320 \text{ V}$$

For $i = 1$, $t_1 = 0.00002$, $v_1 = 53.320$

$$v_2 = v_1 + f(t_1, v_1)h$$

$$= 53.320 + f(0.00002, 53.320)0.00002$$

$$= 53.320 + \frac{1}{150 \times 10^{-6}} \left\{ -0.1 + \max\left(\frac{\begin{array}{c}|18\cos(120\pi(0.00002))| \\ -2-(53.320)\end{array}}{0.04}, 0 \right) \right\} 0.00002$$

$$= 53.320 + (-0.000015000)0.00002$$

$$= 53.307 \text{ V}$$

v_2 is the approximate voltage at

$$t = t_2 = t_1 + h = 0.00002 + 0.00002 = 0.00004 \text{ s}$$

$$v(0.00004) \approx v_2 = 53.307 \text{ V}$$

Figure 1 compares the exact solution of $v(0.00004) = 15.974 \text{ V}$ with the numerical solution from Euler's method for the step size of $h = 0.00004 \text{ s}$.

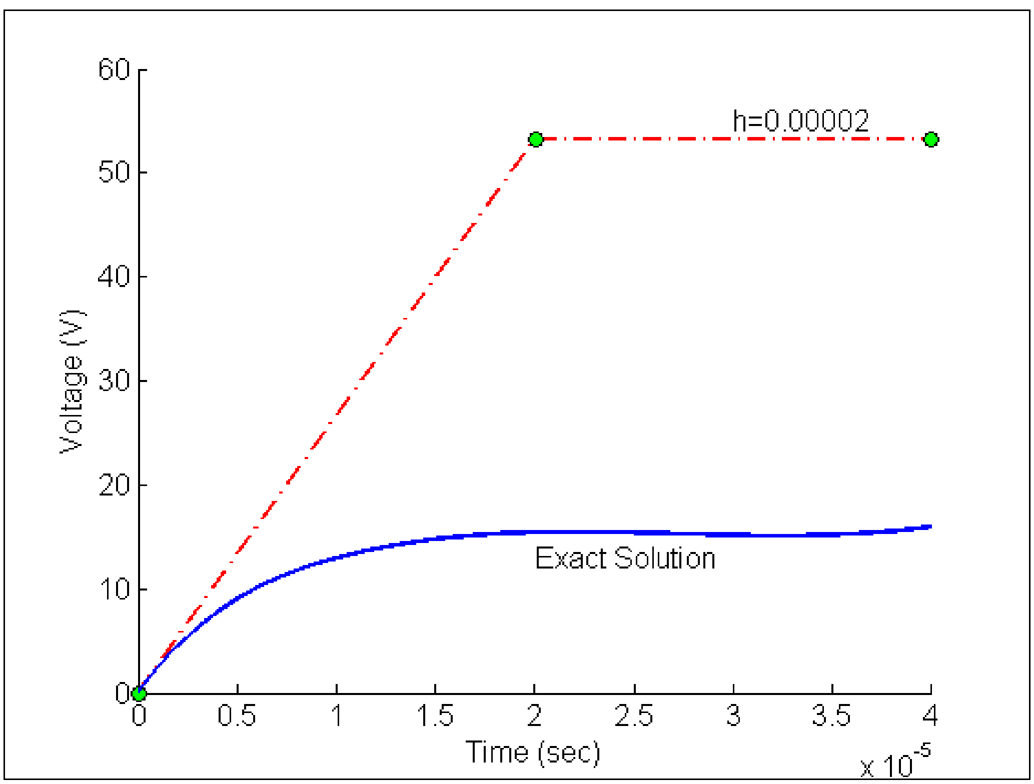

Figure 1 Comparing exact and Euler's method.

The problem was solved again using smaller step sizes. The results are given below in Table 1.

Table 1 Voltage at 0.00004 seconds as a function of step size, h.

Step size, h	$v(0.00004)$	E_t	$\lvert\in_t\rvert\,\%$
0.00004	106.64	−90.667	567.59
0.00002	53.307	−37.333	233.71
0.00001	26.640	−10.666	66.771
0.000005	15.996	−0.021991	0.13766
0.0000025	15.993	−0.019125	0.11972

Figure 2 shows how the voltage varies as a function of time for different step sizes.

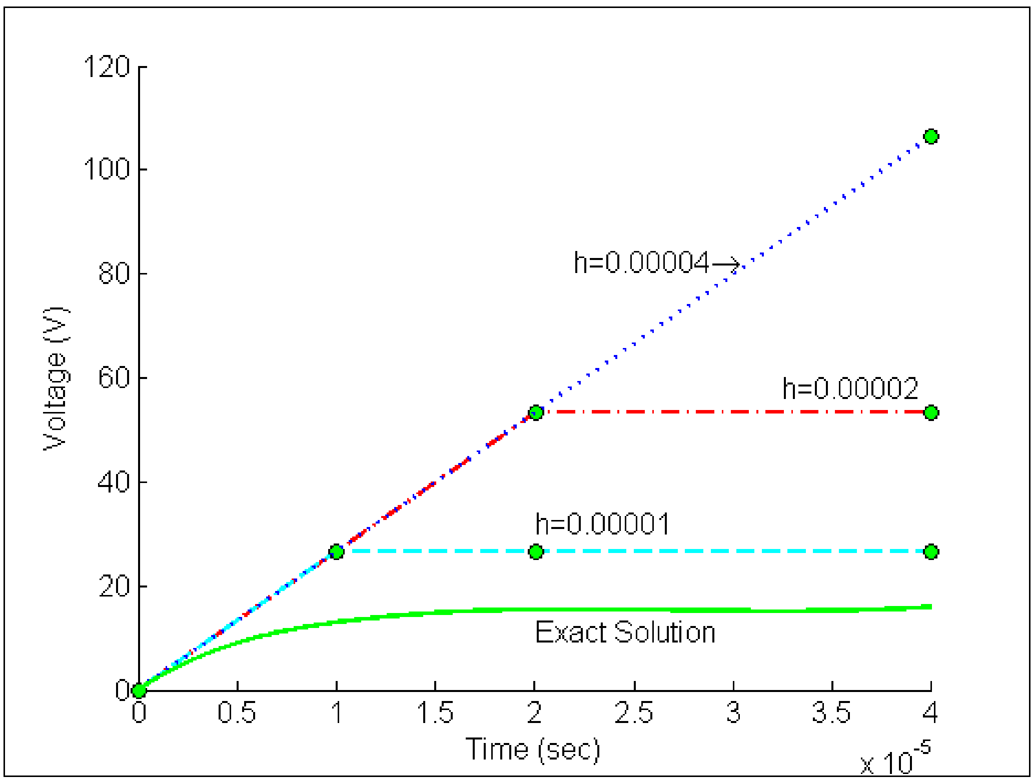

Figure 2 Comparison of Euler's method with exact solution for different step sizes.

While the values of the calculated voltage at $t = 0.00004\,\text{s}$ as a function of step size are plotted in Figure 3.

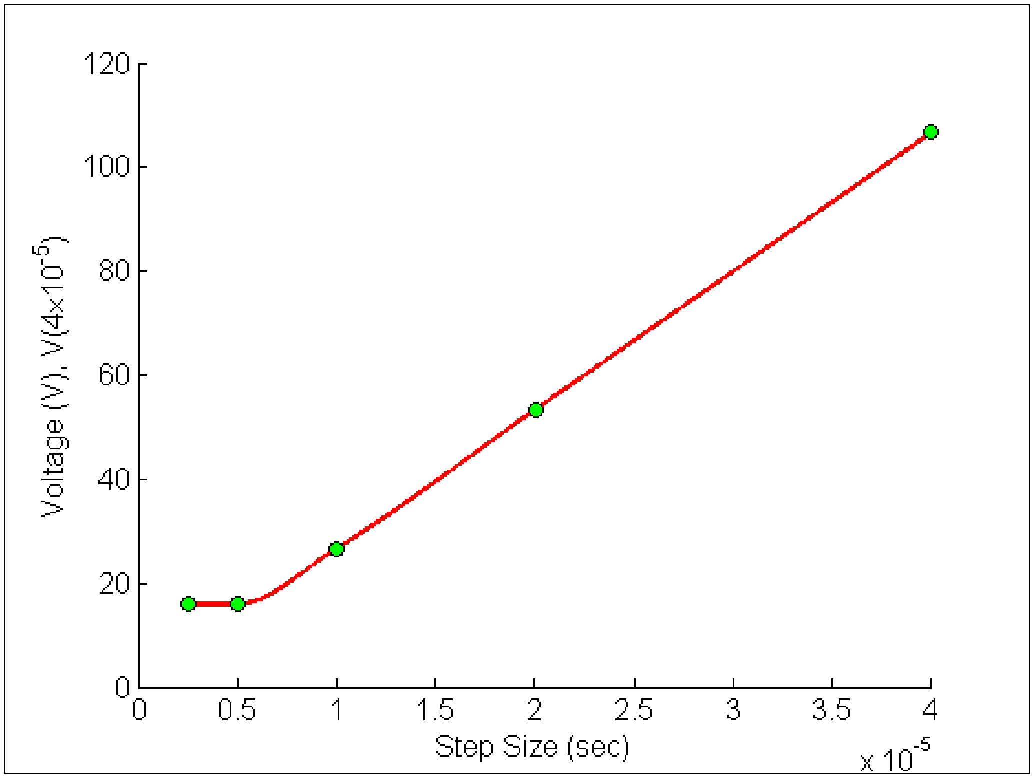

Figure 3 Effect of step size in Euler's method.