

Chapter 03.04

Newton-Raphson Method of Solving a Nonlinear Equation – More Examples

Electrical Engineering

Example 1

Thermistors are temperature-measuring devices based on the principle that the thermistor material exhibits a change in electrical resistance with a change in temperature. By measuring the resistance of the thermistor material, one can then determine the temperature. For a 10K3A Betatherm thermistor,

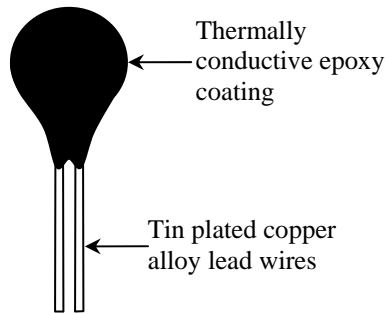


Figure 1 A typical thermistor.

the relationship between the resistance R of the thermistor and the temperature is given by

$$\frac{1}{T} = 1.129241 \times 10^{-3} + 2.341077 \times 10^{-4} \ln(R) + 8.775468 \times 10^{-8} \{\ln(R)\}^3$$

where T is in Kelvin and R is in ohms.

A thermistor error of no more than $\pm 0.01^\circ\text{C}$ is acceptable. To find the range of the resistance that is within this acceptable limit at 19°C , we need to solve

$$\frac{1}{19.01 + 273.15} = 1.129241 \times 10^{-3} + 2.341077 \times 10^{-4} \ln(R) + 8.775468 \times 10^{-8} \{\ln(R)\}^3$$

and

$$\frac{1}{18.99 + 273.15} = 1.129241 \times 10^{-3} + 2.341077 \times 10^{-4} \ln(R) + 8.775468 \times 10^{-8} \{\ln(R)\}^3$$

Use the Newton-Raphson method of finding roots of equations to find the resistance R at 18.99°C . Conduct three iterations to estimate the root of the above equation. Find the absolute relative approximate error at the end of each iteration and the number of significant digits at least correct at the end of each iteration.

Solution

Solving

$$\frac{1}{18.99 + 273.15} = 1.129241 \times 10^{-3} + 2.341077 \times 10^{-4} \ln(R) + 8.775468 \times 10^{-8} \{\ln(R)\}^3$$

we get

$$f(R) = 2.341077 \times 10^{-4} \ln(R) + 8.775468 \times 10^{-8} \{\ln(R)\}^3 - 2.293775 \times 10^{-3}$$

$$f'(R) = \frac{2.341077 \times 10^{-4} + 2.6326404 \times 10^{-7} \{\ln(R)\}^2}{R}$$

Let us take the initial guess of the root of $f(R) = 0$ as $R_0 = 15000$.

Iteration 1

The estimate of the root is

$$\begin{aligned} R_1 &= R_0 - \frac{f(R_0)}{f'(R_0)} \\ &= 15000 - \frac{\left(2.341077 \times 10^{-4} \ln(15000) + 8.775468 \times 10^{-8} \{\ln(15000)\}^3 - 2.293775 \times 10^{-3} \right)}{\frac{2.341077 \times 10^{-4} + 2.6326404 \times 10^{-7} \{\ln(15000)\}^2}{15000}} \\ &= 15000 - \frac{3.5383 \times 10^{-5}}{1.7230 \times 10^{-8}} \\ &= 15000 - 2053.5 \\ &= 12946 \end{aligned}$$

The absolute relative approximate error $|\epsilon_a|$ at the end of Iteration 1 is

$$\begin{aligned} |\epsilon_a| &= \left| \frac{R_1 - R_0}{R_1} \right| \times 100 \\ &= \left| \frac{12946 - 15000}{12946} \right| \times 100 \\ &= 15.862\% \end{aligned}$$

The number of significant digits at least correct is 0, as you need an absolute relative approximate error of less than 5% for one significant digit to be correct in your result.

Iteration 2

The estimate of the root is

$$R_2 = R_1 - \frac{f(R_1)}{f'(R_1)}$$

$$\begin{aligned}
&= 12946 - \frac{\left(\begin{array}{l} 2.341077 \times 10^{-4} \ln(12946) + 8.775468 \times 10^{-8} \{\ln(12946)\}^3 \\ - 2.293775 \times 10^{-3} \end{array} \right)}{\frac{2.341077 \times 10^{-4} + 2.6326404 \times 10^{-7} \{\ln(12946)\}^2}{12946}} \\
&= 12946 - \frac{-2.6140 \times 10^{-6}}{1.9906 \times 10^{-8}} \\
&= 12946 - (-131.32) \\
&= 13078
\end{aligned}$$

The absolute relative approximate error $|\epsilon_a|$ at the end of Iteration 2 is

$$\begin{aligned}
|\epsilon_a| &= \left| \frac{R_2 - R_1}{R_2} \right| \times 100 \\
&= \left| \frac{13078 - 12946}{13078} \right| \times 100 \\
&= 1.0041\%
\end{aligned}$$

The number of significant digits at least correct is 1, as the absolute relative approximate error is less than 5% .

Iteration 3

The estimate of the root is

$$\begin{aligned}
R_3 &= R_2 - \frac{f(R_2)}{f'(R_2)} \\
&= 13078 - \frac{\left(\begin{array}{l} 2.341077 \times 10^{-4} \ln(13078) + 8.775468 \times 10^{-8} \{\ln(13078)\}^3 \\ - 2.293775 \times 10^{-3} \end{array} \right)}{\frac{2.341077 \times 10^{-4} + 2.6326404 \times 10^{-7} \{\ln(13078)\}^2}{13078}} \\
&= 13078 - \frac{-1.2914 \times 10^{-8}}{1.9710 \times 10^{-8}} \\
&= 13078 - (-0.65519) \\
&= 13078
\end{aligned}$$

The absolute relative approximate error $|\epsilon_a|$ at the end of Iteration 3 is

$$\begin{aligned}
|\epsilon_a| &= \left| \frac{R_3 - R_2}{R_3} \right| \times 100 \\
&= \left| \frac{13078 - 13078}{13078} \right| \times 100 \\
&= 0.0050097\%
\end{aligned}$$

Hence the number of significant digits at least correct is given by the largest value of m for which

$$|\epsilon_a| \leq 0.5 \times 10^{2-m}$$

$$0.0050097 \leq 0.5 \times 10^{2-m}$$

$$0.010019 \leq 10^{2-m}$$

$$\log(0.010019) \leq 2 - m$$

$$m \leq 2 - \log(0.010019) = 3.9992$$

So

$$m = 3$$

The number of significant digits at least correct in the estimated root 13078 is 3.

NONLINEAR EQUATIONS

| | |
|----------|---|
| Topic | Newton-Raphson Method-More Examples |
| Summary | Examples of Newton-Raphson Method |
| Major | Electrical Engineering |
| Authors | Autar Kaw |
| Date | August 7, 2009 |
| Web Site | http://numericalmethods.eng.usf.edu |
