Chapter 07.03
Simpson's 1/3 Rule for Integration-More Examples
Civil Engineering

Example 1

The concentration of benzene at a critical location is given by

$$c = 1.75\left[erfc(0.6560) + e^{32.73} erfc(5.758)\right]$$

where

$$erfc(x) = \int_{\infty}^{x} e^{-z^2} dz$$

So in the above formula

$$erfc(0.6560) = \int_{\infty}^{0.6560} e^{-z^2} dz$$

Since e^{-z^2} decays rapidly as $z \to \infty$, we will approximate

$$erfc(0.6560) = \int_{5}^{0.6560} e^{-z^2} dz$$

a) Use Simpson's 1/3 Rule to find the value of $erfc(0.6560)$.
b) Find the true error, E_t, for part (a).
c) Find the absolute relative true error for part (a).

Solution

a) $$erfc(0.6560) \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

$$a = 5$$
$$b = 0.6560$$
$$\frac{a+b}{2} = 2.8280$$
$$f(z) = e^{-z^2}$$
$$f(5) = e^{-5^2}$$
$$= 1.3888 \times 10^{-11}$$
$$f(0.6560) = e^{-0.6560^2}$$
$$= 0.65029$$

$$f(2.8280) = e^{-2.8280^2}$$
$$= 3.3627 \times 10^{-4}$$
$$erfc(0.6560) = \left(\frac{b-a}{6}\right)\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$
$$= \left(\frac{0.6560-5}{6}\right)[f(5) + 4f(2.8280) + f(0.6560)]$$
$$= \left(\frac{-4.3440}{6}\right)\left[1.3888 \times 10^{-11} + 4\left(3.3627 \times 10^{-4}\right) + 0.65029\right]$$
$$= -0.47178$$

b) The exact value of the above integral cannot be found. For calculating the true error and relative true error, we assume the value obtained by adaptive numerical integration using Maple as the exact value.

$$erfc(0.6560) = \int_{5}^{0.6560} e^{-z^2} dz$$
$$= -0.31333$$

so the true error is

$$E_t = True\ Value - Approximate\ Value$$
$$= -0.31333 - (-0.47178)$$
$$= 0.15846$$

c) The absolute relative true error, $|\in_t|$, would then be

$$|\in_t| = \left|\frac{\text{True Error}}{\text{True Value}}\right| \times 100$$
$$= \left|\frac{0.15846}{-0.31333}\right| \times 100$$
$$= 50.573\,\%$$

Example 2

The concentration of benzene at a critical location is given by

$$c = 1.75\left[erfc(0.6560) + e^{32.73} erfc(5.758)\right]$$

where

$$erfc(x) = \int_{\infty}^{x} e^{-z^2} dz$$

So in the above formula

$$erfc(0.6560) = \int_{\infty}^{0.6560} e^{-z^2} dz$$

Since e^{-z^2} decays rapidly as $z \to \infty$, we will approximate

$$erfc(0.6560) = \int_{5}^{0.6560} e^{-z^2}\,dz$$

a) Use four segment Simpson's 1/3 Rule to find the value of $erfc(0.6560)$.
b) Find the true error, E_t, for part (a).
c) Find the absolute relative true error for part (a).

Solution

a) $$erfc(0.6560) = \frac{b-a}{3n}\left[f(z_0) + 4\sum_{\substack{i=1\\ i=odd}}^{n-1} f(z_i) + 2\sum_{\substack{i=2\\ i=even}}^{n-2} f(z_i) + f(z_n)\right]$$

$$n = 4$$
$$a = 5$$
$$b = 0.6560$$
$$h = \frac{b-a}{n}$$
$$= \frac{0.6560-5}{4}$$
$$= -1.0860$$
$$f(z) = e^{-z^2}$$

So

$$f(z_0) = f(5)$$
$$f(5) = e^{-5^2}$$
$$= 1.3888\times 10^{-11}$$
$$f(z_1) = f(5-1.0860)$$
$$= f(3.9140)$$
$$f(3.9140) = e^{-3.9140^2}$$
$$= 2.2226\times 10^{-7}$$
$$f(z_2) = f(3.9140-1.0860)$$
$$= f(2.8280)$$
$$f(2.8280) = e^{-2.8280^2}$$
$$= 3.3627\times 10^{-4}$$
$$f(z_3) = f(2.8280-1.0860)$$
$$= f(1.7420)$$
$$f(1.7420) = e^{-1.7420^2}$$
$$= 0.048096$$

$$f(z_4) = f(z_n)$$
$$= f(0.6560)$$
$$f(0.6560) = e^{-0.6560^2}$$
$$= 0.65029$$

$$erfc(0.6560) = \frac{b-a}{3n}\left[f(z_0) + 4\sum_{\substack{i=1\\ i=odd}}^{n-1} f(z_i) + 2\sum_{\substack{i=2\\ i=even}}^{n-2} f(z_i) + f(z_n) \right]$$
$$= \frac{0.6560-5}{3(4)}\left[f(5) + 4\sum_{\substack{i=1\\ i=odd}}^{3} f(z_i) + 2\sum_{\substack{i=2\\ i=even}}^{2} f(z_i) + f(0.6560) \right]$$
$$= \frac{-4.3440}{12}\left[f(5) + 4f(z_1) + 4f(z_3) + 2f(z_2) + f(0.6560) \right]$$
$$= \frac{-4.3440}{12}\begin{bmatrix} f(5) + 4f(3.9140) + 4f(1.7420) \\ + 2f(2.8280) + f(0.6560) \end{bmatrix}$$
$$= \frac{-4.3440}{12}\begin{bmatrix} 1.3888\times10^{-11} + 4(2.2226\times10^{-7}) + 4(0.048096) \\ + 2(3.3627\times10^{-4}) + 0.65029 \end{bmatrix}$$
$$= -0.30529$$

b) The exact value of the above integral cannot be found. For calculating the true error and relative true error, we assume the value obtained by adaptive numerical integration using Maple as the exact value.

$$erfc(0.6560) = \int_{5}^{0.6560} e^{-z^2}\,dz$$
$$= -0.31333$$

so the true error is

$$E_t = True\ Value - Approximate\ Value$$
$$= -0.31333 - (-0.30529)$$
$$= -0.0080347$$

c) The absolute relative true error, $|\in_t|$, would then be

$$|\in_t| = \left|\frac{\text{True Error}}{\text{True Value}}\right| \times 100$$
$$= \left|\frac{-0.0080347}{-0.31333}\right| \times 100$$
$$= 2.5643\,\%$$

Table 1 Values of Simpson's 1/3 Rule for Example 2 with multiple segments.

n	Approximate Value	E_t	$\lvert \in_t \rvert$ %
2	−0.47178	0.15846	50.573
4	−0.30529	−0.0080347	2.5643
6	−0.30678	−0.0065444	2.0887
8	−0.31110	−0.0022249	0.71009
10	−0.31248	−0.00084868	0.27086