Chapter 05.03
Newton's Divided Difference Interpolation – More Examples
Chemical Engineering

Example 1

To find how much heat is required to bring a kettle of water to its boiling point, you are asked to calculate the specific heat of water at $61°C$. The specific heat of water is given as a function of time in Table 1.

Table 1 Specific heat of water as a function of temperature.

Temperature, T $(°C)$	Specific heat, C_p $\left(\frac{J}{kg-°C}\right)$
22	4181
42	4179
52	4186
82	4199
100	4217

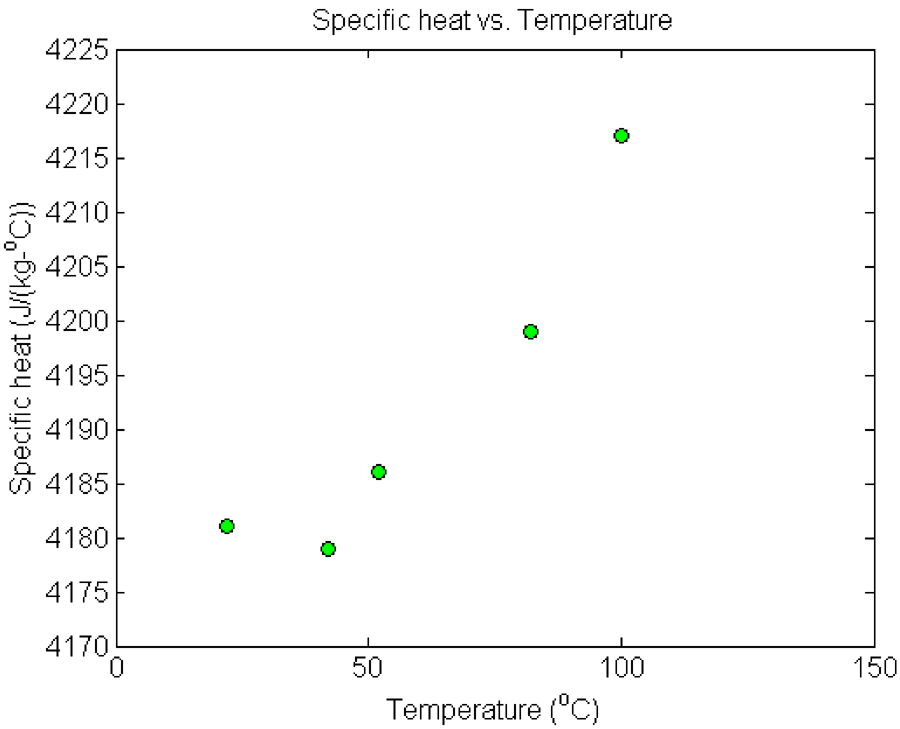

Figure 1 Specific heat of water vs. temperature.

Determine the value of the specific heat at $T = 61°\text{C}$ using Newton's divided difference method of interpolation and a first order polynomial.

Solution

For linear interpolation, the specific heat is given by

$$C_p(T) = b_0 + b_1(T - T_0)$$

Since we want to find the velocity at $T = 61°\text{C}$, and we are using a first order polynomial we need to choose the two data points that are closest to $T = 61°\text{C}$ that also bracket $T = 61°\text{C}$ to evaluate it. The two points are $T = 52$ and $T = 82$.

Then

$$T_0 = 52, \quad C_p(T_0) = 4186$$

$$T_1 = 82, \quad C_p(T_1) = 4199$$

gives

$$\begin{aligned} b_0 &= C_p(T_0) \\ &= 4186 \end{aligned}$$

$$\begin{aligned} b_1 &= \frac{C_p(T_1) - C_p(T_0)}{T_1 - T_0} \\ &= \frac{4199 - 4186}{82 - 52} \\ &= 0.43333 \end{aligned}$$

Hence

$$C_p(T) = b_0 + b_1(T - T_0)$$
$$= 4186 + 0.43333(T - 52), \qquad 52 \le T \le 82$$

At $T = 61$,

$$C_p(61) = 4186 + 0.43333(61 - 52)$$
$$= 4189.9 \frac{\text{J}}{\text{kg} - {}^\circ\text{C}}$$

If we expand

$$C_p(T) = 4186 + 0.43333(T - 52), \qquad 52 \le T \le 82$$

we get

$$C_p(T) = 4163.5 + 0.43333T, \qquad 52 \le T \le 82$$

and this is the same expression as obtained in the direct method.

Example 2

To find how much heat is required to bring a kettle of water to its boiling point, you are asked to calculate the specific heat of water at 61°C. The specific heat of water is given as a function of time in Table 2.

Table 2 Specific heat of water as a function of temperature.

Temperature, T $(^\circ\text{C})$	Specific heat, C_p $\left(\frac{\text{J}}{\text{kg} - {}^\circ\text{C}}\right)$
22	4181
42	4179
52	4186
82	4199
100	4217

Determine the value of the specific heat at $T = 61^\circ\text{C}$ using Newton's divided difference method of interpolation and a second order polynomial. Find the absolute relative approximate error for the second order polynomial approximation.

Solution

For quadric interpolation, the specific heat is given by

$$C_p(T) = b_0 + b_1(T - T_0) + b_2(T - T_0)(T - T_1)$$

Since we want to find the specific heat at $T = 61^\circ\text{C}$, and we are using a second order polynomial, we need to choose the three data points that are closest to $T = 61^\circ\text{C}$ that also bracket $T = 61^\circ\text{C}$ to evaluate it. The three points are $T_0 = 42,\ T_1 = 52,$ and $T_2 = 82$.

Then

$$T_0 = 42,\ C_p(T_0) = 4179$$

$$T_1 = 52,\ \ C_p(T_1) = 4186$$
$$T_2 = 82,\ \ C_p(T_2) = 4199$$

gives

$$b_0 = C_p(T_0)$$
$$= 4179$$
$$b_1 = \frac{C_p(T_1) - C_p(T_0)}{T_1 - T_0}$$
$$= \frac{4186 - 4179}{52 - 42}$$
$$= 0.7$$
$$b_2 = \frac{\dfrac{C_p(T_2) - C_p(T_1)}{T_2 - T_1} - \dfrac{C_p(T_1) - C_p(T_0)}{T_1 - T_0}}{T_2 - T_0}$$
$$= \frac{\dfrac{4199 - 4186}{82 - 52} - \dfrac{4186 - 4179}{52 - 42}}{82 - 42}$$
$$= \frac{0.43333 - 0.7}{40}$$
$$= -6.6667 \times 10^{-3}$$

Hence

$$C_p(T) = b_0 + b_1(T - T_0) + b_2(T - T_0)(T - T_1)$$
$$= 4179 + 0.7(T - 42) - 6.6667 \times 10^{-3}(T - 42)(T - 52), \qquad 42 \le T \le 82$$

At $T = 61$,

$$C_p(61) = 4179 + 0.7(61 - 42) - 6.6667 \times 10^{-3}(61 - 42)(61 - 52)$$
$$= 4191.2 \frac{\text{J}}{\text{kg} - {}^\circ\text{C}}$$

The absolute relative approximate error $|\in_a|$ obtained between the results from the first and second order polynomial is

$$|\in_a| = \left|\frac{4191.2 - 4189.9}{4191.2}\right| \times 100$$
$$= 0.030063\%$$

If we expand

$$C_p(T) = 4179 + 0.7(T - 42) - 6.6667 \times 10^{-3}(T - 42)(T - 52), \qquad 42 \le T \le 82$$

we get

$$C_p(T) = 4135.0 + 1.3267T - 6.6667 \times 10^{-3}T^2, \qquad 42 \le T \le 82$$

This is the same expression obtained by the direct method.

Example 3

To find how much heat is required to bring a kettle of water to its boiling point, you are asked to calculate the specific heat of water at $61°C$. The specific heat of water is given as a function of time in Table 3.

Table 3 Specific heat of water as a function of temperature.

Temperature, T $(°C)$	Specific heat, C_p $\left(\frac{J}{kg-°C}\right)$
22	4181
42	4179
52	4186
82	4199
100	4217

Determine the value of the specific heat at $T = 61°C$ using Newton's divided difference method of interpolation and a third order polynomial. Find the absolute relative approximate error for the third order polynomial approximation.

Solution

For a third order polynomial, the specific heat profile is given by

$$C_p(T) = b_0 + b_1(T - T_0) + b_2(T - T_0)(T - T_1) + b_3(T - T_0)(T - T_1)(T - T_2)$$

Since we want to find the specific heat at $T = 61°C$, and we are using a third order polynomial, we need to choose the four data points that are closest to $T = 61°C$ that also bracket $T = 61°C$. The four data points are $T_0 = 42,\ T_1 = 52,\ T_2 = 82$ and $T_3 = 100$.
(Choosing the four points as $T_0 = 22$, $T_1 = 42$, $T_2 = 52$ and $T_3 = 82$ is equally valid.)

$$T_0 = 42, \quad C_p(T_0) = 4179$$
$$T_1 = 52, \quad C_p(T_1) = 4186$$
$$T_2 = 82, \quad C_p(T_2) = 4199$$
$$T_3 = 100, \quad C_p(T_3) = 4217$$

then

$$\begin{aligned} b_0 &= C_p[T_0] \\ &= C_p(T_0) \\ &= 4179 \end{aligned}$$

$$\begin{aligned} b_1 &= C_p[T_1, T_0] \\ &= \frac{C_p(T_1) - C_p(T_0)}{T_1 - T_0} \\ &= \frac{4186 - 4179}{52 - 42} \end{aligned}$$

$$= 0.7$$

$$b_2 = C_p[T_2, T_1, T_0]$$

$$= \frac{C_p[T_2, T_1] - C_p[T_1, T_0]}{T_2 - T_0}$$

$$C_p[T_2, T_1] = \frac{C_p(T_2) - C_p(T_1)}{T_2 - T_1}$$

$$= \frac{4199 - 4186}{82 - 52}$$

$$= 0.43333$$

$$C_p[T_1, T_0] = 0.7$$

$$b_2 = \frac{C_p[T_2, T_1] - C_p[T_1, T_0]}{T_2 - T_0}$$

$$= \frac{0.43333 - 0.7}{82 - 42}$$

$$= -6.6667 \times 10^{-3}$$

$$b_3 = C_p\left[T_3, T_2, T_1, T_0\right]$$

$$= \frac{C_p[T_3, T_2, T_1] - C_p[T_2, T_1, T_0]}{T_3 - T_0}$$

$$C_p[T_3, T_2, T_1] = \frac{C_p[T_3, T_2] - C_p[T_2, T_1]}{T_3 - T_1}$$

$$C_p[T_3, T_2] = \frac{C_p(T_3) - C_p(T_2)}{T_3 - T_2}$$

$$= \frac{4217 - 4199}{100 - 82}$$

$$= 1$$

$$C_p[T_2, T_1] = \frac{C_p(T_2) - C_p(T_1)}{T_2 - T_1}$$

$$= \frac{4199 - 4186}{82 - 52}$$

$$= 0.43333$$

$$C_p[T_3, T_2, T_1] = \frac{C_p[T_3, T_2] - C_p[T_2, T_1]}{T_3 - T_1}$$

$$= \frac{1 - 0.43333}{100 - 52}$$

$$= 0.011806$$

$$C_p[T_2, T_1, T_0] = -6.6667 \times 10^{-3}$$

$$b_3 = C_p[T_3,T_2,T_1,T_0]$$
$$= \frac{C_p[T_3,T_2,T_1] - C_p[T_2,T_1,T_0]}{T_3 - T_0}$$
$$= \frac{0.011806 + 6.6667\times 10^{-3}}{100 - 42}$$
$$= 3.1849\times 10^{-4}$$

Hence

$$C_p(T) = b_0 + b_1(T-T_0) + b_2(T-T_0)(T-T_1) + b_3(T-T_0)(T-T_1)(T-T_2)$$
$$= 4179 + 0.7(T-42) - 6.6667\times 10^{-3}(T-42)(T-52)$$
$$+ 3.1849\times 10^{-4}(T-42)(T-52)(T-82), \qquad 42 \le T \le 100$$

At $T = 61$,

$$C_p(61) = 4179 + 0.7(61-42) - 6.6667\times 10^{-3}(61-42)(61-52)$$
$$+ 3.1849\times 10^{-4}(61-42)(61-52)(61-82)$$
$$= 4190.0\,\frac{\text{J}}{\text{kg} - {}^\circ\text{C}}$$

The absolute relative approximate error $|\in_a|$ obtained between the results from the second and third order polynomial is

$$|\in_a| = \left|\frac{4190.0 - 4191.2}{4190.0}\right| \times 100$$
$$= 0.027295\%$$

If we expand

$$C_p(T) = 4179 + 0.7(T-42) - 6.6667\times 10^{-3}(T-42)(T-52)$$
$$+ 3.1849\times 10^{-4}(T-42)(T-52)(T-82), \qquad 42 \le T \le 100$$

we get

$$C_p(T) = 4078.0 + 4.4771T - 0.06272T^2 + 3.1849\times 10^{-4}T^3,\ 42 \le T \le 100$$

This is the same expression as obtained in the direct method.

INTERPOLATION	
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Summary	Examples of Newton's divided difference interpolation.
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