Numerical Methods

Multi Dimensional Direct Search Methods - Theory

http://nm.mathforcollege.com

For more details on this topic

- Go to http://nm.mathforcollege.com
- Click on Keyword
- Click on Multi Dimensional Direct Search Methods

You are free

- to **Share** – to copy, distribute, display and perform the work
- to **Remix** – to make derivative works

Under the following conditions

- **Attribution** — You must attribute the work in the manner specified by the author or licensor (but not in any way that suggests that they endorse you or your use of the work).
- **Noncommercial** — You may not use this work for commercial purposes.
- **Share Alike** — If you alter, transform, or build upon this work, you may distribute the resulting work only under the same or similar license to this one.

Multi Dimensional Direct Search Methods Method-Overview

- Obvious approach is to enumerate all possible solutions and find the min or the max.
- Very generally applicable but ccomputationally complex
- Direct search methods are open
- A good initial estimate of the solution is required
- The objective function need not be differentiable

Coordinate Cycling Method

- Starts from an initial point and looks for an optimal solution along each coordinate direction iteratively.
- For a function with two independent variables *x* and *y*, starting at an initial point (x_0,y_0), the first iteration will first move along direction (1, 0) until an optimal solution is found for the function .
- The next search involves searching along the direction (0,1) to determine the optimal value for the function.
- Once searches in all directions are completed, the process is repeated in the next iteration and iterations continue until convergence occurs.
- The search along each coordinate direction can be conducted using anyone of the one-dimensional search techniques previously covered.

Multi Dimensional Direct Search

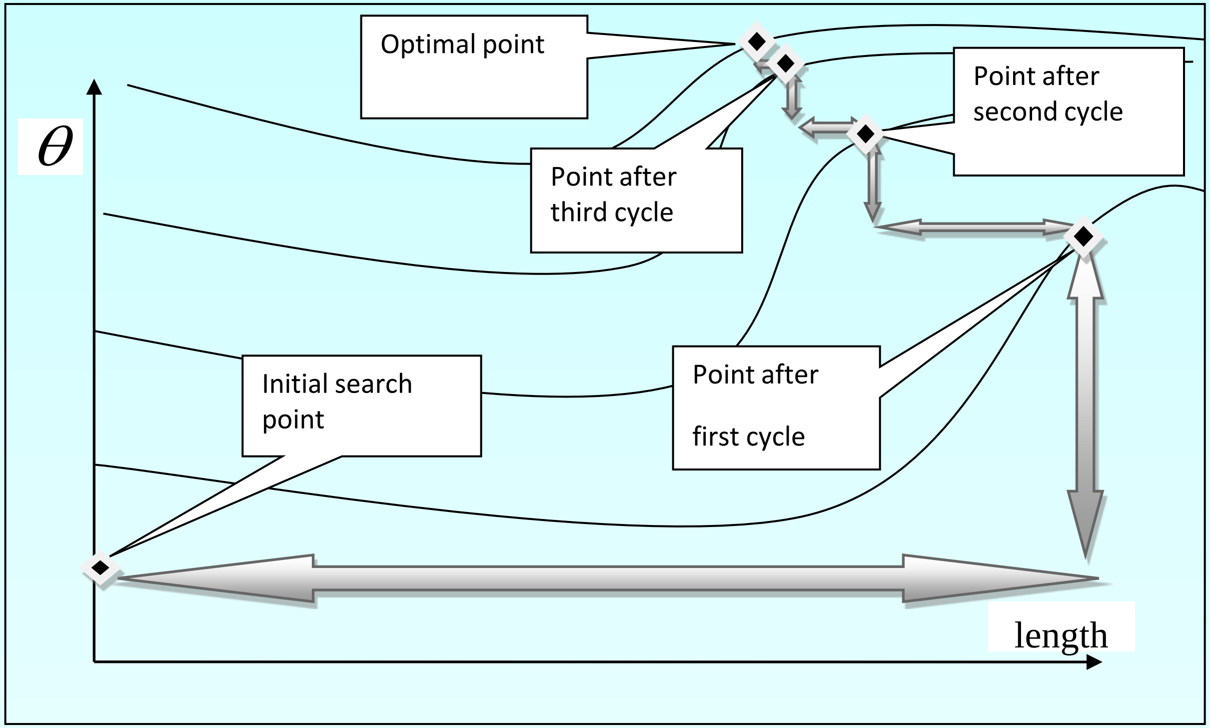


Figure 1: Visual Representation of a Multidimensional Search

THE END

http://nm.mathforcollege.com

Acknowledgement

This instructional power point brought to you by Numerical Methods for STEM undergraduate

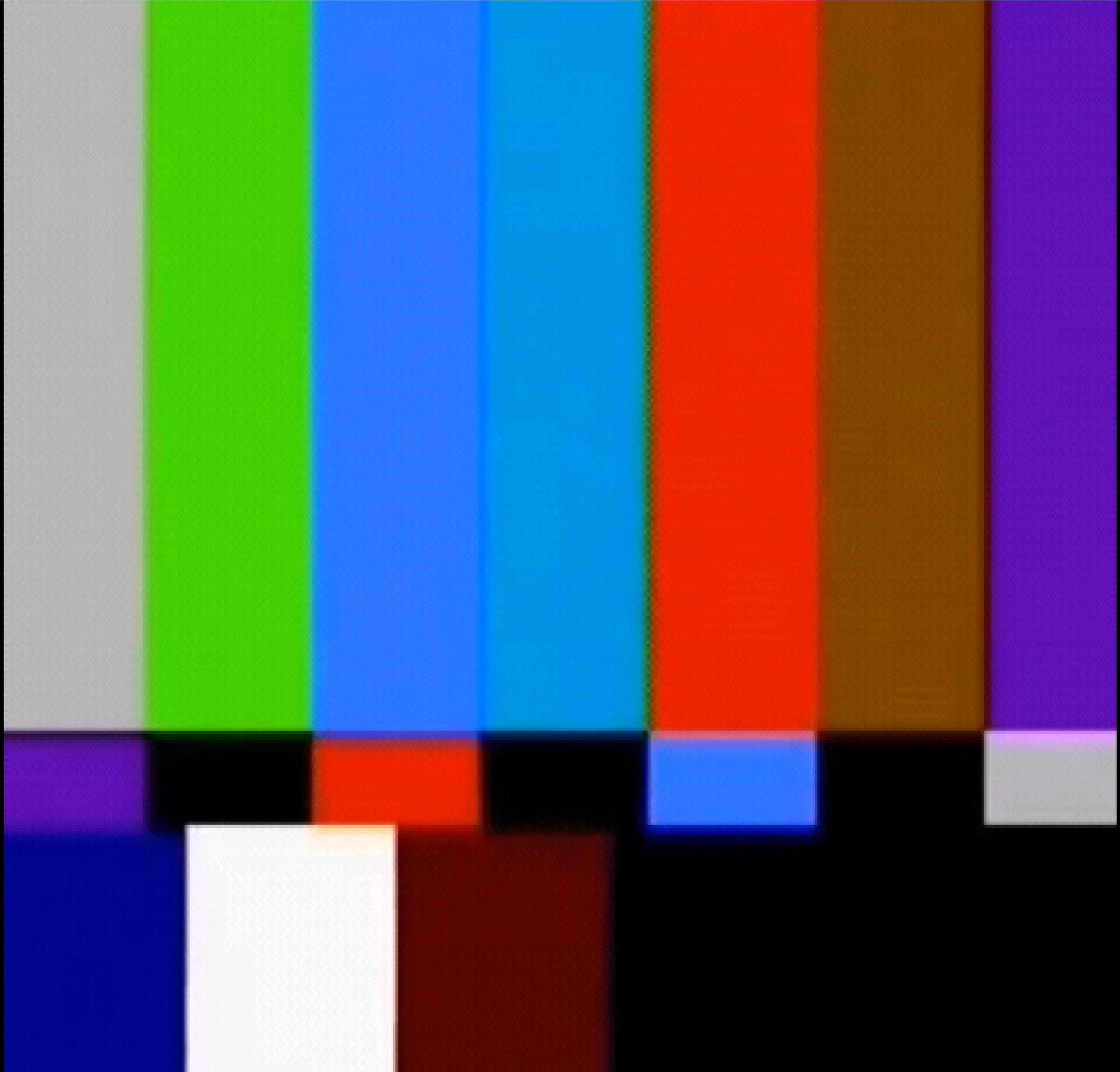
http://nm.mathforcollege.com

Committed to bringing numerical methods to the undergraduate

For instructional videos on other topics, go to

http://nm.mathforcollege.com

This material is based upon work supported by the National Science Foundation under Grant # 0717624. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.



Numerical Methods

Multi Dimensional Direct Search Methods - Example

http://nm.mathforcollege.com

For more details on this topic

- Go to http://nm.mathforcollege.com
- Click on Keyword
- Click on Multi Dimensional Direct Search Methods

You are free

- to **Share** – to copy, distribute, display and perform the work
- to **Remix** – to make derivative works

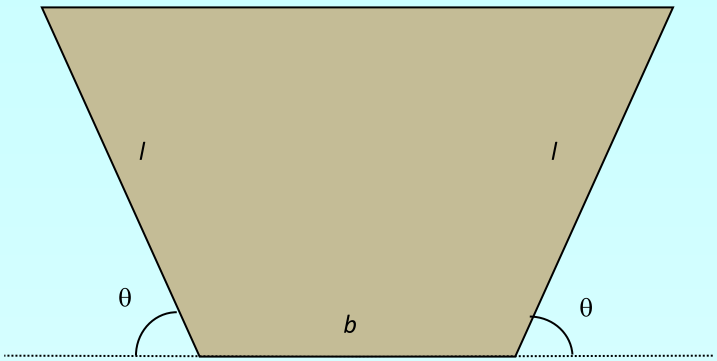
Old Dominion University

USF University of South Florida

Under the following conditions

- **Attribution** — You must attribute the work in the manner specified by the author or licensor (but not in any way that suggests that they endorse you or your use of the work).
- **Noncommercial** — You may not use this work for commercial purposes.
- **Share Alike** — If you alter, transform, or build upon this work, you may distribute the resulting work only under the same or similar license to this one.

Example



The cross-sectional area A of a gutter with base length *b* and edge length of *l* is given by

$$A = \frac{1}{2}(b + b + 2l\cos\theta)l\sin\theta$$

Assuming that the width of material to be bent into the gutter shape is $6(6 = b + 2l)$, find the angle θ and edge length l which maximizes the cross-sectional area of the gutter.

Notes :

- To get the maximum cross-sectional area $0 \le \theta \le \frac{\pi}{2} \cong 1.5708$
- To have a physical meaning, $l_{max} \le 3$ (otherwise "b" will have a negative value!)

Solution

- Recognizing that the base length *b* can be expressed as $b = 6 - 2l$, we can re-write the area function as

$$f(l,\theta) = (6 - 2l + l\cos\theta)l\sin\theta$$

- Use $(0, \frac{\pi}{6} = 0.5236)$ as the initial estimate of the solution and use Golden Search method to determine optimal solution in each dimension.

- To use the golden search method we will use $l = 0 \rightarrow 3$ as the lower and upper bounds for the search region

Solution Cont.

Iteration 1 along (1,0)

$$f(l,\theta = 0.5236rad) = [6 - 2l + l\cos(0.5236)]l\sin(0.5236)$$

Iteration	x_l	x_u	x_1	x_2	$f(x_1)$	$f(x_2)$	ε
1	0.0000	3.0000	1.8541	1.1459	3.6143	2.6941	3.0000
2	1.1459	3.0000	2.2918	1.8541	3.8985	3.6143	1.8541
3	1.8541	3.0000	2.5623	2.2918	3.9655	3.8985	1.1459
4	2.2918	3.0000	2.7295	2.5623	3.9654	3.9655	0.7082
5	2.2918	2.7295	2.5623	2.4590	3.9655	3.9497	0.4377
6	2.4590	2.7295	2.6262	2.5623	3.9692	3.9655	0.2705
7	2.5623	2.7295	2.6656	2.6262	3.9692	3.9692	0.1672
8	2.5623	2.6656	2.6262	2.6018	3.9692	3.9683	0.1033
9	2.6018	2.6656	2.6412	2.6262	3.9694	3.9692	0.0639
10	2.6262	2.6656	2.6506	2.6412	3.9694	3.9694	0.0395

The maximum area of 3.6964 is obtained at point $(2.6459, 0.5236)$ by using either Golden Section Method (see above table) or analytical method (set $\frac{df}{dl} = 0 \Rightarrow l = 2.6459$).

Solution Cont.

Iteration 1 along (0,1)

$$f(l=2.6459,\theta)=[6-2\times 2.6459+2.6459\cos\theta]\times 2.6459\sin\theta$$

Iteration	x_l	x_u	x_1	x_2	$f(x_1)$	$f(x_2)$	ε
1	0.0000	$\boxed{1.5714=\frac{\pi}{2}}$	0.9712	0.6002	4.8084	4.3215	1.5714
2	0.6002	1.5714	1.2005	0.9712	4.1088	4.8084	0.9712
3	0.6002	1.2005	0.9712	0.8295	4.8084	4.8689	0.6002
4	0.6002	0.9712	0.8295	0.7419	4.8689	4.7533	0.3710
5	0.7419	0.9712	0.8836	0.8295	4.8816	4.8689	0.2293
6	0.8295	0.9712	0.9171	0.8836	4.8672	4.8816	0.1417
7	0.8295	0.9171	0.8836	0.8630	4.8816	4.8820	0.0876
8	0.8295	0.8836	0.8630	0.8502	4.8820	4.8790	0.0541
9	0.8502	0.8836	0.8708	0.8630	4.8826	4.8820	0.0334

The maximum area of 4.8823 is obtained at point $(2.6459, 0.87)$,by using either Golden Section Method (see above table) or analytical method (set $\frac{df}{d\theta}=0 \Rightarrow \theta=0.87$).

Solution Cont.

- Since this is a two-dimensional search problem, the two searches along the two dimensions completes the first iteration.
- In the next iteration we return to the first dimension for which we conducted a search and start the second iteration with a search along this dimension.
- After the fifth cycle, the optimal solution of (2.0016, 1.0420) with an area of 5.1960 is obtained.
- The optimal solution to the problem is exactly 60 degrees which is 1.0472 radians and an edge and base length of 2 inches. The area of the gutter at this point is 5.1962.

THE END

http://nm.mathforcollege.com

Acknowledgement

This instructional power point brought to you by Numerical Methods for STEM undergraduate

http://nm.mathforcollege.com

Committed to bringing numerical methods to the undergraduate

For instructional videos on other topics, go to

http://nm.mathforcollege.com

This material is based upon work supported by the National Science Foundation under Grant # 0717624. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.