Measuring Errors

Major: All Engineering Majors

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Measuring Errors

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Why measure errors?

- 1) To determine the accuracy of numerical results.
- 2) To develop stopping criteria for iterative algorithms.

True Error

 Defined as the difference between the true value in a calculation and the approximate value found using a numerical method etc.

True Error = True Value – Approximate Value

Example—True Error

The derivative, f'(x) of a function f(x) can be approximated by the equation,

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

If $f(x) = 7e^{0.5x}$ and h = 0.3

a) Find the approximate value of f'(2)

b) True value of f'(2)

c) True error for part (a)

Solution: a) For x = 2 and h = 0.3 $f'(2) \approx \frac{f(2+0.3) - f(2)}{0.3}$ $=\frac{f(2.3)-f(2)}{0.3}$ $=\frac{7e^{0.5(2.3)}-7e^{0.5(2)}}{0.3}$ $=\frac{22.107 - 19.028}{0.3} = 10.263$

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Solution:

b) The exact value of f'(2) can be found by using our knowledge of differential calculus.

 $f(x) = 7e^{0.5x}$ $f'(x) = 7 \times 0.5 \times e^{0.5x}$ $= 3.5e^{0.5x}$ So the true value of f'(2) is $f'(2) = 3.5e^{0.5(2)}$ = 9.5140

True error is calculated as E_t = True Value – Approximate Value = 9.5140-10.263 = -0.722

Relative True Error

Defined as the ratio between the true error, and the true value.

Relative True Error $(\in_t) = \frac{\text{True Error}}{\text{True Value}}$

Example—Relative True Error

Following from the previous example for true error, find the relative true error for $f(x) = 7e^{0.5x}$ at f'(2)with h = 0.3

From the previous example,

 $E_t = -0.722$

Relative True Error is defined as

 $\epsilon_t = \frac{\text{True Error}}{\text{True Value}}$ $= \frac{-0.722}{9.5140} = -0.075888$ as a percentage, $\epsilon_t = -0.075888 \times 100\% = -7.5888\%$

Approximate Error

- What can be done if true values are not known or are very difficult to obtain?
- Approximate error is defined as the difference between the present approximation and the previous approximation.

Approximate Error (E_a) = Present Approximation – Previous Approximation

Example—Approximate Error

For $f(x) = 7e^{0.5x}$ at x = 2 find the following,

a)
$$f'(2)$$
 using $h = 0.3$

b) f'(2) using h = 0.15

c) approximate error for the value of $f'^{(2)}$ for part b) Solution:

a) For
$$x = 2$$
 and $h = 0.3$
 $f'(x) \approx \frac{f(x+h) - f(x)}{h}$
 $f'(2) \approx \frac{f(2+0.3) - f(2)}{0.3}$

Example (cont.)
Solution: (cont.)

$$= \frac{f(2.3) - f(2)}{0.3}$$

$$= \frac{7e^{0.5(2.3)} - 7e^{0.5(2)}}{0.3}$$

$$= \frac{22.107 - 19.028}{0.3} = 10.263$$
b) For $x = 2$ and $h = 0.15$
 $f'(2) \approx \frac{f(2+0.15) - f(2)}{0.15}$
 $= \frac{f(2.15) - f(2)}{0.15}$

Solution: (cont.) = $\frac{7e^{0.5(2.15)} - 7e^{0.5(2)}}{0.15}$ = $\frac{20.50 - 19.028}{0.15}$ = 9.8800

c) So the approximate error, E_a is

$$E_a$$
 = Present Approximation – Previous Approximation
= 9.8800 – 10.263

= -0.38300

Relative Approximate Error

 Defined as the ratio between the approximate error and the present approximation.

Relative Approximate Error $(\in_a) =$

Approximate Error

Present Approximation

Example—Relative Approximate Error

For $f(x) = 7e^{0.5x}$ at x = 2, find the relative approximate error using values from h = 0.3 and h = 0.15

Solution:

From Example 3, the approximate value of f'(2) = 10.263using h = 0.3 and f'(2) = 9.8800 using h = 0.15

$$E_a$$
 = Present Approximation – Previous Approximation

= 9.8800 - 10.263

= -0.38300

Solution: (cont.)

$$\epsilon_a = \frac{\text{Approximate Error}}{\text{Present Approximation}}$$
$$= \frac{-0.38300}{9.8800} = -0.038765$$

as a percentage,

 $\epsilon_a = -0.038765 \times 100\% = -3.8765\%$

Absolute relative approximate errors may also need to be calculated,

 $|\epsilon_a| = |-0.038765| = 0.038765$ or 3.8765%

How is Absolute Relative Error used as a stopping criterion?

If $|e_a| \le e_s$ where e_s is a pre-specified tolerance, then no further iterations are necessary and the process is stopped.

If at least *m* significant digits are required to be correct in the final answer, then $|\epsilon_a| \le 0.5 \times 10^{2-m}\%$

Table of Values

For $f(x) = 7e^{0.5x}$ at x = 2 with varying step size, h

h	f'(2)	$ \epsilon_a $	т
0.3	10.263	N/A	0
0.15	9.8800	3.877%	1
0.10	9.7558	1.273%	1
0.01	9.5378	2.285%	1
0.001	9.5164	0.2249%	2

Additional Resources

For all resources on this topic such as digital audiovisual lectures, primers, textbook chapters, multiple-choice tests, worksheets in MATLAB, MATHEMATICA, MathCad and MAPLE, blogs, related physical problems, please visit

http://numericalmethods.eng.usf.edu/topics/measuring _errors.html

THE END

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