

## Chapter 08.00B

# Physical Problem for Ordinary Differential Equations Chemical Engineering

Soap is prepared through a reaction known as saponification. In saponification, tallow (fats from animals such as cattle) or vegetable fat (e.g. coconut) is reacted with potassium or sodium hydroxide to produce glycerol and fatty acid salt known as “soap”. The soap is separated from the glycerol through precipitation by the addition of sodium chloride. Water layer on top of the mixture that contains dissolved sodium chloride is drawn-off the mixture as a waste. This method of soap making is still being practiced in many villages in the developing countries where the price of mass produced soap maybe too expensive for the average villager.

Two chemical engineering students used knowledge of saponification acquired in their organic chemistry class to organize and produce “home made” soap. The local ordinance requires that the minimum concentration level for sodium chloride waste in any liquid that is discharged into the environment must not exceed 11.00g/L. Sodium chloride laden liquid water is the major waste of the process. The company has only one 15-liter tank for waste storage. On filling the waste tank, the tank contained 15 liters of water and 750 grams of sodium chloride. To continue production and meet local ordinance, it is desired to pump in fresh water into the tank at the rate of 2.0 liters per minute while waste salt water containing 25 grams of salt per liter is added at the rate of 1.5 liters per minute. To keep the solution level at 15 liters, 3.5 liters per minute of the waste is discharged. A sketch Figure 1 of the flows is given below where A represents the waste stream from the process, B is the fresh water stream and C is the discharge stream to the environment. Here, it is assumed that as the two streams, A and B enter into the tank, instantaneously the chloride concentration in the tank changes to the exit concentration,  $x_1$ .

The material (sodium chloride) balance on the tank system can be written as

$$\text{Accumulation} = \text{input} - \text{output} + \text{removal by reaction} \quad (1)$$

Noting that no chemical reaction occurs in the storage tank (i.e. the third term on the right hand side of (1) is zero), the above equation can be written as

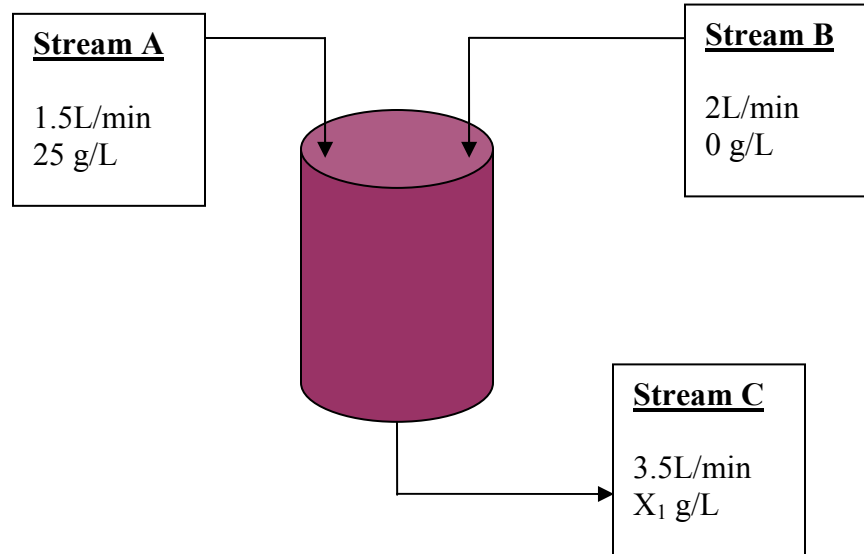
$$\frac{dx_1}{dt} = (25\text{g/L})(1.5\text{L/min}) + (0\text{g/L})(2\text{L/min}) - (x_1\text{g/L})(3.5\text{L/min}) + 0 \quad (2)$$

Simplifying Equation (2), we obtain

$$\frac{dx_1}{dt} + 3.5x_1 = 37.5 \quad (3)$$

For the initial conditions of the ordinary differential equations in Equation (3), recall that at  $t = 0$ , the salt concentration in the tank was given as 750 g/15L (50g/L), that is,

$$t = 0, x_1(0) = \frac{750}{15} = 50\text{g/L} \quad (4)$$



**Figure 1** Sketch of flow for a saponification process.

## QUESTIONS

1. Using Equations (3) and (4), numerically determine how the concentration of salt being discharged changes with time.
2. Plot the solution obtained.
3. How long did it take to achieve the minimum required local ordinance? At steady state, what is the concentration of salt being discharged from this local soap factory?

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### Ordinary Differential Equation

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Topic	Ordinary Differential Equations
Summary	Sodium chloride waste control while making soap
Major	Chemical Engineering
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