Multiple-Choice Test

Chapter 04.11
Cholesky and LDL^T Decomposition

1. For a given set of simultaneous linear equations

$$[A][x]=[b]$$

where

$$[A]=\begin{bmatrix} 2 & -1 & 0 & 0.5 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0.5 & 0 & -1 & 1 \end{bmatrix} \text{ and } [b]=\begin{bmatrix} -0.5 \\ 5 \\ -5 \\ 1.5 \end{bmatrix}$$

The Cholesky factorized matrix $[U]$ can be computed as

(A) $\begin{bmatrix} 1.414 & 0.7071 & 0 & 0.3536 \\ 0 & 1.225 & -0.8165 & 0.2041 \\ 0 & 0 & 1.155 & -0.7217 \\ 0 & 0 & 0 & 0.5590 \end{bmatrix}$

(B) $\begin{bmatrix} 1.414 & -0.7071 & 0 & 0.3536 \\ 0 & 1.225 & -0.8165 & 0.2041 \\ 0 & 0 & 1.155 & -0.7217 \\ 0 & 0 & 0 & 0.5590 \end{bmatrix}$

(C) $\begin{bmatrix} 1.414 & 0.7071 & 0 & -0.3536 \\ 0 & 1.225 & -0.8165 & 0.2041 \\ 0 & 0 & 1.155 & -0.7217 \\ 0 & 0 & 0 & 0.5590 \end{bmatrix}$

(D) $\begin{bmatrix} 1.414 & 0.7071 & 0 & 0.3536 \\ 0 & 1.225 & -0.8165 & 0.2041 \\ 0 & 0 & -1.155 & -0.7217 \\ 0 & 0 & 0 & 0.5590 \end{bmatrix}$

2. For a given set of simultaneous linear equations

$$[A][x]=[b]$$

where

$$[A]=\begin{bmatrix} 2 & -1 & 0 & 0.5 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0.5 & 0 & -1 & 1 \end{bmatrix} \text{ and } [b]=\begin{bmatrix} -0.5 \\ 5 \\ -5 \\ 1.5 \end{bmatrix}$$

the forward solution vector $[y]$ can be computed as

(A) $\vec{y}^T = \{0.5363, 38.784, -15.877, 0.5590\}$
(B) $\vec{y}^T = \{0.5363, -15.877, 38.784, 0.5590\}$
(C) $\vec{y}^T = \{-3.536, -1.5877, 3.878, 0.5590\}$
(D) $\vec{y}^T = \{-0.3536, 3.8784, -1.5877, -0.5590\}$

3. For a given set of simultaneous linear equations

$$[A][x]=[b]$$

where

$$[A]=\begin{bmatrix} 2 & -1 & 0 & 0.5 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0.5 & 0 & -1 & 1 \end{bmatrix} \text{ and } [b]=\begin{bmatrix} -0.5 \\ 5 \\ -5 \\ 1.5 \end{bmatrix}$$

The backward solution vector $[x]$ can be computed as

(A) $\vec{x}^T = \{1, 2, -2, -1\}$
(B) $\vec{x}^T = \{1, 2, 2, -1\}$
(C) $\vec{x}^T = \{-1, 2, -2, 1\}$
(D) $\vec{x}^T = \{1, 2, 2, 1\}$

4. The determinant of

$$[A]=\begin{bmatrix} 2 & -1 & 0 & 0.5 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0.5 & 0 & -1 & 1 \end{bmatrix}$$

most nearly is

(A) -5
(B) 5
(C) -50
(D) 1.25

5. Based on the given matrix $[A]$, and assuming the reordering algorithm will produce the following mapping IPERM (new equation #) = {old equation #}, such as

$$IPERM\begin{pmatrix}1\\2\\3\\4\end{pmatrix}=\begin{pmatrix}3\\1\\4\\2\end{pmatrix},$$

the non zero off-diagonal term A(old row 4, old column 1) = 0.5 will move to the following new location of the new matrix $[A^*]$

(A) A^*(new row 3, new column 1)
(B) A^*(new row 1, new column 3)
(C) A^*(new row 3, new column 2)
(D) A^*(new row 2, new column 2)

6. Based on the given matrix $[A]$, and the given reordering mapping

$$IPERM\begin{pmatrix}1\\2\\3\\4\end{pmatrix}=\begin{pmatrix}3\\1\\4\\2\end{pmatrix},$$

the non-zero diagonal term $A(4,4)=1$ will move to the following new location of the new matrix $[A^*]$

(A) $A^*(1,1)=1$
(B) $A^*(2,2)=1$
(C) $A^*(3,3)=1$
(D) $A^*(4,4)=1$

7. For a given set of simultaneous linear equations, and using LDL^T algorithm,

$$[A][x]=[b]$$

where

$$[A]=\begin{bmatrix} 2 & -1 & 0 & 0.5 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0.5 & 0 & -1 & 1 \end{bmatrix} \text{ and } [b]=\begin{bmatrix} 2 \\ -1 \\ 1 \\ 0.5 \end{bmatrix},$$

the lower triangular matrix $[L]$ can be computed as

(A) $[L]=\begin{bmatrix} 1 & 0 & 0 & 0 \\ -0.5 & 1 & 0 & 0 \\ 0 & -0.6667 & 1 & 0 \\ 0.25 & 0.1667 & -0.625 & 1 \end{bmatrix}$

(B) $[L]=\begin{bmatrix} 1 & 0 & 0 & 0 \\ -0.5 & 1 & 0 & 0 \\ 0 & 0.6667 & 1 & 0 \\ 0.25 & 0.1667 & -0.625 & 1 \end{bmatrix}$

(C) $[L]=\begin{bmatrix} 1 & 0 & 0 & 0 \\ -0.5 & 1 & 0 & 0 \\ 0 & -0.6667 & 1 & 0 \\ 0.25 & 0.1667 & 0.625 & 1 \end{bmatrix}$

(D) $[L]=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 \\ 0 & -0.6667 & 1 & 0 \\ 0.25 & 0.1667 & -0.625 & 1 \end{bmatrix}$

8. For a given set of simultaneous linear equations, and using LDL^T algorithm,

$$[A][x]=[b]$$

where

$$[A]=\begin{bmatrix} 2 & -1 & 0 & 0.5 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0.5 & 0 & -1 & 1 \end{bmatrix} \text{ and } [b]=\begin{bmatrix} 2 \\ -1 \\ 1 \\ 0.5 \end{bmatrix},$$

the diagonal matrix $[D]$ can be computed as:

(A) $[D]=\begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 1.3333 & 0 \\ 0 & 0 & 0 & 0.3125 \end{bmatrix}$

(B) $[D]=\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -1.5 & 0 & 0 \\ 0 & 0 & 1.3333 & 0 \\ 0 & 0 & 0 & 0.3125 \end{bmatrix}$

(C) $[D]=\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 1.3333 & 0 \\ 0 & 0 & 0 & -0.3125 \end{bmatrix}$

(D) $[D]=\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 1.3333 & 0 \\ 0 & 0 & 0 & 0.3125 \end{bmatrix}$

9. For a given set of simultaneous linear equations, and using LDL^T algorithm,

$$[A][x]=[b]$$

where

$$[A]=\begin{bmatrix} 2 & -1 & 0 & 0.5 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0.5 & 0 & -1 & 1 \end{bmatrix} \text{ and } [b]=\begin{bmatrix} 2 \\ -1 \\ 1 \\ 0.5 \end{bmatrix},$$

the forward solution for the unknown vector $[z]$ in $[L][z]=[b]$ can be computed as

(A) $\{z\}^T=\{-2, 0, 1, 0.625\}$

(B) $\{z\}^T=\{2, 0, 1, 0.625\}$

(C) $\{z\}^T=\{2, 0, -1, 0.625\}$

(D) $\{z\}^T=\{2, 0, 1, -0.625\}$

10. For a given set of simultaneous linear equations, and using LDL^T algorithm,

$$[A][x]=[b]$$

where

$$[A]=\begin{bmatrix} 2 & -1 & 0 & 0.5 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0.5 & 0 & -1 & 1 \end{bmatrix} \text{ and } [b]=\begin{bmatrix} 2 \\ -1 \\ 1 \\ 0.5 \end{bmatrix},$$

the diagonal scaling solution for the unknown vector $[y]$ in $[D][y]=[z]$ can be computed as:

(A) $\{y\}^T=\{-1, 0, 0.75, 2\}$

(B) $\{y\}^T=\{1, 0, -0.75, 2\}$

(C) $\{y\}^T=\{1, 0, 0.75, -2\}$

(D) $\{y\}^T=\{1, 0, 0.75, 2\}$

11. For a given set of simultaneous linear equations, and using LDL^T algorithm,

$$[A][x]=[b]$$

where

$$[A]=\begin{bmatrix} 2 & -1 & 0 & 0.5 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0.5 & 0 & -1 & 1 \end{bmatrix} \text{ and } [b]=\begin{bmatrix} 2 \\ -1 \\ 1 \\ 0.5 \end{bmatrix},$$

the backward solution for the original unknown vector $[x]$, in $[L]^T[x]=[y]$, can be computed as

(A) $\{x\}^T=\{1, 1, 2, 2\}$

(B) $\{x\}^T=\{2, 1, 2, 1\}$

(C) $\{x\}^T=\{1, 1, 2, 1\}$

(D) $\{x\}^T=\{2, 2, 2, 1\}$

12. Given the following 6×6 matrix $[A]$, which is assumed to be SPD:

$$[A]=\begin{bmatrix} \times & 0 & \times & 0 & \times & 0 \\ & \times & 0 & \times & 0 & 0 \\ & & \times & 0 & \times & \times \\ & & & \times & 0 & 0 \\ & Sym & & & \times & 0 \\ & & & & & \times \end{bmatrix}$$

where $\times$ = a nonzero value (given)

0 = a zero value (given)

Based on the numerically factorized formulas shown in Equations 6-7 of Chapter 04.11, or even more helpful information as indicated in Figure 1 of Chapter 04.11, and given

* = a nonzero value (computed, at the same location as the original nonzero value of $[A]$)

0 = a zero value

F = a nonzero fill-in-term (computed)

and

$U(5,6)=F$

$A(5,6)=0$

the symbolically factorized upper-triangular matrix can be obtained as

(A) $$[U]=\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ & * & 0 & * & 0 & 0 \\ & & * & 0 & * & * \\ & & & * & 0 & 0 \\ & & & & * & F \\ & & & & & * \end{bmatrix}$$

(B) $$[U]=\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ & * & 0 & F & 0 & 0 \\ & & * & 0 & * & * \\ & & & * & 0 & 0 \\ & & & & * & F \\ & & & & & * \end{bmatrix}$$

(C) $$[U]=\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ & * & 0 & F & 0 & 0 \\ & & * & 0 & F & * \\ & & & * & 0 & 0 \\ & & & & * & F \\ & & & & & * \end{bmatrix}$$

(D) $$[U]=\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ & * & 0 & F & 0 & 0 \\ & & * & 0 & F & F \\ & & & * & 0 & 0 \\ & & & & * & F \\ & & & & & * \end{bmatrix}$$