Chapter 07.02
Trapezoidal Rule for Integration-More Examples
Mechanical Engineering

Example 1

A trunnion of diameter $12.363''$ has to be cooled from a room temperature of $80°\text{F}$ before it is shrink fit into a steel hub (Figure 1).

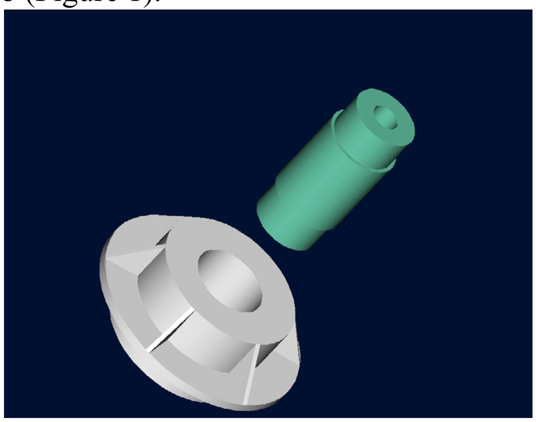

Figure 1 Trunnion to be slid through the hub after contracting.

The equation that gives the diametric contraction, in inches of the trunnion in dry-ice/alcohol (boiling temperature is $-108°\text{F}$) is given by:

$$\Delta D = 12.363 \int_{80}^{-108} \left(-1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.015 \times 10^{-6}\right) dT$$

a) Use single segment Trapezoidal rule to find the contraction.
b) Find the true error, E_t, for part (a).
c) Find the absolute relative true error for part (a).

Solution

a) $I \approx (b-a)\left[\frac{f(a)+f(b)}{2}\right]$, where

$$a = 80$$
$$b = -108$$
$$f(T) = 12.363\left(-1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.015 \times 10^{-6}\right)$$
$$f(80) = 12.363\left(-1.2278 \times 10^{-11}(80)^2 + 6.1946 \times 10^{-9}(80) + 6.015 \times 10^{-6}\right)$$
$$= 7.9519 \times 10^{-5}$$
$$f(-108) = 12.363\left(-1.2278 \times 10^{-11}(-108)^2 + 6.1946 \times 10^{-9}(-108) + 6.015 \times 10^{-6}\right)$$

$$= 6.4322 \times 10^{-5}$$

$$I \approx (-108 - 80)\left[\frac{7.9519 \times 10^{-5} + 6.4322 \times 10^{-5}}{2}\right]$$

$$\approx -0.013521 \text{ in}$$

b) The exact value of the above integral is

$$\Delta D = 12.363 \int_{80}^{-108} \left(-1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.015 \times 10^{-6}\right) dT$$

$$= -0.013689 \text{ in}$$

so the true error is

$$E_t = \text{True Value} - \text{Approximate Value}$$

$$= -0.013689 - (-0.013521)$$

$$= -0.00016810$$

c) The absolute relative true error, $|\in_t|$, would then be

$$|\in_t| = \left|\frac{\text{True Error}}{\text{True Value}}\right| \times 100\,\%$$

$$= \left|\frac{-0.00016810}{-0.013689}\right| \times 100\,\%$$

$$= 1.2280\,\%$$

Example 2

A trunnion of diameter $12.363''$ has to be cooled from a room temperature of $80°\text{F}$ before it is shrink fit into a steel hub (Figure 1). The equation that gives the diametric contraction, in inches of the trunnion in dry-ice/alcohol (boiling temperature is $-108°\text{F}$) is given by:

$$\Delta D = 12.363 \int_{80}^{-108} \left(-1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.015 \times 10^{-6}\right) dT$$

Use two segment Trapezoidal rule to find the contraction.

a) Find the true error, E_t, for part (a).
b) Find the absolute relative true error for part (a).

Solution

a) The solution using 2-segment Trapezoidal rule is

$$I \approx \frac{b-a}{2n}\left[f(a) + 2\left\{\sum_{i=1}^{n-1} f(a+ih)\right\} + f(b)\right]$$

$$n = 2$$

$$a = 80$$

$$b = -108$$

$$h = \frac{b-a}{n}$$

$$= \frac{-108-80}{2}$$

$$= -94$$

$$I \approx \frac{-108-80}{2(2)}\left[f(80)+2\left\{\sum_{i=1}^{2-1} f(a+ih)\right\}+f(-108)\right]$$

$$\approx \frac{-188}{4}\left[f(80)+2f(80+1\times(-94))+f(-108)\right]$$

$$\approx \frac{-188}{4}\left[f(80)+2f(-14)+f(-108)\right]$$

$$\approx -47\left[7.9519\times10^{-5}+2\left(7.3262\times10^{-5}\right)+6.4322\times10^{-5}\right]$$

$$\approx -0.013647 \text{ in}$$

b) The exact value of the above integral is

$$\Delta D = 12.363\int_{80}^{-108}\left(-1.2278\times10^{-11}T^2+6.1946\times10^{-9}T+6.015\times10^{-6}\right)dT$$

$$= -0.013689 \text{ in}$$

so the true error is

$$E_t = \text{True Value} - \text{Approximate Value}$$

$$= -0.013689-(-0.013647)$$

$$= -0.000042026 \text{ in}$$

c) The absolute relative true error, $|\in_t|$, would then be

$$|\in_t| = \left|\frac{\text{True Error}}{\text{True Value}}\right|\times 100\ \%$$

$$= \left|\frac{-0.000042026}{-0.013689}\right|\times 100\ \%$$

$$= 0.30700\ \%$$

Table 1 Values obtained using multiple-segment Trapezoidal rule for

$$\Delta D = 12.363\int_{80}^{-108}\left(-1.2278\times10^{-11}T^2+6.1946\times10^{-9}T+6.015\times10^{-6}\right)dT$$

n	Value	E_t	$\lvert\in_t\rvert$%	$\lvert\in_a\rvert$%
1	−0.013521	−0.00016810	1.2280	---
2	−0.013647	-4.2026×10^{-5}	0.30700	0.92328
3	−0.013670	-1.8678×10^{-5}	0.13644	0.16825
4	−0.013679	-1.0506×10^{-5}	0.076750	0.059740
5	−0.013682	-6.7241×10^{-6}	0.049120	0.027644
6	−0.013684	-4.6695×10^{-6}	0.034111	0.015014
7	−0.013686	-3.4307×10^{-6}	0.025061	0.0090522
8	−0.013687	-2.6266×10^{-6}	0.019188	0.0058749