Chapter 07.05
Romberg Rule for Integration-More Examples
Mechanical Engineering

Example 1

A trunnion of diameter $12.363''$ has to be cooled from a room temperature of $80°\text{F}$ before it is shrink fit into a steel hub (Figure 1).

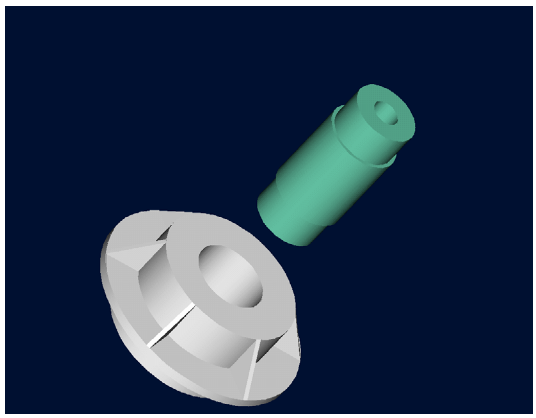

Figure 1 Trunnion to be slid through the hub after contracting.

The equation that gives the diametric contraction, in inches of the trunnion in dry-ice/alcohol (boiling temperature is $-108°\text{F}$) is given by:

$$\Delta D = 12.363 \int_{80}^{-108} \left(-1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.015 \times 10^{-6}\right) dT$$

Table 1 Values obtained for Trapezoidal rule.

n	Trapezoidal Rule
1	−0.013536
2	−0.013630
4	−0.013679
8	−0.013687

a) Use Romberg's rule to find the contraction. Use the 2-segment and 4-segment Trapezoidal rule results given in Table 1.
b) Find the true error, E_t, for part (a).
c) Find the absolute relative true error for part (a).

Solution

a) $I_2 = -0.013630$ in

$I_4 = -0.013679$ in

Using Richardson's extrapolation formula for Trapezoidal rule

$$TV \approx I_{2n} + \frac{I_{2n} - I_n}{3}$$

and choosing $n = 2$,

$$TV \approx I_4 + \frac{I_4 - I_2}{3}$$

$$\approx -0.013679 + \frac{-0.013679 - (-0.013630)}{3}$$

$$\approx -0.013670 \text{ in}$$

b) The exact value of the above integral is

$$\Delta D = 12.363 \int_{80}^{-108} \left(-1.2278\times10^{-11}T^2 + 6.1946\times10^{-9}T + 6.015\times10^{-6}\right)dT$$

$$= -0.013689 \text{ in}$$

so the true error is

$$E_t = \text{True Value} - \text{Approximate Value}$$

$$= -0.013689 - (-0.013670)$$

$$= 5.5212\times10^{-7} \text{ in}$$

c) The absolute relative true error, $|\in_t|$, would then be

$$|\in_t| = \left|\frac{\text{True Error}}{\text{True Value}}\right| \times 100\,\%$$

$$= \left|\frac{5.5212\times10^{-7}}{-0.013689}\right| \times 100\,\%$$

$$= 0.0040332\,\%$$

Table 2 shows the Richardson's extrapolation results using 1, 2, 4, 8 segments. Results are compared with those of Trapezoidal rule.

Table 2 Values obtained using Richardson's extrapolation formula for Trapezoidal rule for

$$\Delta D = 12.363 \int_{80}^{-108} \left(-1.2278\times10^{-11}T^2 + 6.1946\times10^{-9}T + 6.015\times10^{-6}\right)dT$$

n	Trapezoidal Rule	$\|\in_t\|$ for Trapezoidal Rule %	Richardson's Extrapolation	$\|\in_t\|$ for Richardson's Extrapolation %
1	−0.013536	1.1177	--	--
2	−0.013630	0.43100	−0.013661	0.20294
4	−0.013679	0.076750	−0.013695	0.045429
8	−0.013687	0.019187	−0.013690	0.0040332

Example 2

A trunnion of diameter $12.363''$ has to be cooled from a room temperature of $80°\text{F}$ before it is shrink fit into a steel hub (Figure 1). The equation that gives the diametric contraction, in inches of the trunnion in dry-ice/alcohol (boiling temperature is $-108°\text{F}$) is given by:

$$\Delta D = 12.363 \int_{80}^{-108} \left(-1.2278\times10^{-11}T^2 + 6.1946\times10^{-9}T + 6.015\times10^{-6}\right)dT$$

Use Romberg's rule to find the contraction. Use the 1, 2, 4, and 8-segment Trapezoidal rule results as given in the Table 1.

Solution

From Table 1, the needed values from original Trapezoidal rule are

$$I_{1,1} = -0.013536 \text{ in}$$

$$I_{1,2} = -0.013630 \text{ in}$$

$$I_{1,3} = -0.013679 \text{ in}$$

$$I_{1,4} = -0.013687 \text{ in}$$

where the above four values correspond to using 1, 2, 4 and 8 segment Trapezoidal rule, respectively. To get the first order extrapolation values,

$$I_{2,1} = I_{1,2} + \frac{I_{1,2} - I_{1,1}}{3}$$

$$= -0.013630 + \frac{-0.013630 - (-0.013536)}{3}$$

$$= -0.013661 \text{ in}$$

Similarly

$$I_{2,2} = I_{1,3} + \frac{I_{1,3} - I_{1,2}}{3}$$

$$= -0.013679 + \frac{-0.013679 - (-0.013630)}{3}$$

$$= -0.013695 \text{ in}$$

$$I_{2,3} = I_{1,4} + \frac{I_{1,4} - I_{1,3}}{3}$$

$$= -0.013687 + \frac{-0.013687 - (-0.013679)}{3}$$

$$= -0.013695 \text{ in}$$

For the second order extrapolation values,

$$I_{3,1} = I_{2,2} + \frac{I_{2,2} - I_{2,1}}{15}$$

$$= -0.013695 + \frac{-0.013695 - (-0.013661)}{15}$$

$$= -0.013698 \text{ in}$$

Similarly

$$
\begin{aligned}
I_{3,2} &= I_{2,3} + \frac{I_{2,3} - I_{2,2}}{15} \\
&= -0.013695 + \frac{-0.013695 - (-0.013695)}{15} \\
&= -0.013690 \text{ in}
\end{aligned}
$$

For the third order extrapolation values,

$$
\begin{aligned}
I_{4,1} &= I_{3,2} + \frac{I_{3,2} - I_{3,1}}{63} \\
&= -0.013690 + \frac{-0.013690 - (-0.013698)}{63} \\
&= -0.013689 \text{ in}
\end{aligned}
$$

Table 3 shows these increased correct values in a tree graph.

Table 3 Improved estimates of value of integral using Romberg integration.

		1st Order	2nd Order	3rd Order
1-segment	–0.013536			
		–0.013661		
			–0.013698	
2-segment	–0.013630			
				–0.013689
		–0.013695		
			–0.013690	
4-segment	–0.013679			
		–0.013695		
8-segment	–0.013687			