Chapter 05.03
Newton's Divided Difference Interpolation – More Examples
Mechanical Engineering

Example 1

For the purpose of shrinking a trunnion into a hub, the reduction of diameter ΔD of a trunnion shaft by cooling it through a temperature change of ΔT is given by

$$\Delta D = D\alpha\Delta T$$

where

$D =$ original diameter (in.)

$\alpha =$ coefficient of thermal expansion at average temperature (in/in/°F)

The trunnion is cooled from $80°\text{F}$ to $-108°\text{F}$, giving the average temperature as $-14°\text{F}$. The table of the coefficient of thermal expansion vs. temperature data is given in Table 1.

Table 1 Thermal expansion coefficient as a function of temperature.

Temperature, T $(°\text{F})$	Thermal Expansion Coefficient, α (in/in/°F)
80	6.47×10^{-6}
0	6.00×10^{-6}
–60	5.58×10^{-6}
–160	4.72×10^{-6}
–260	3.58×10^{-6}
–340	2.45×10^{-6}

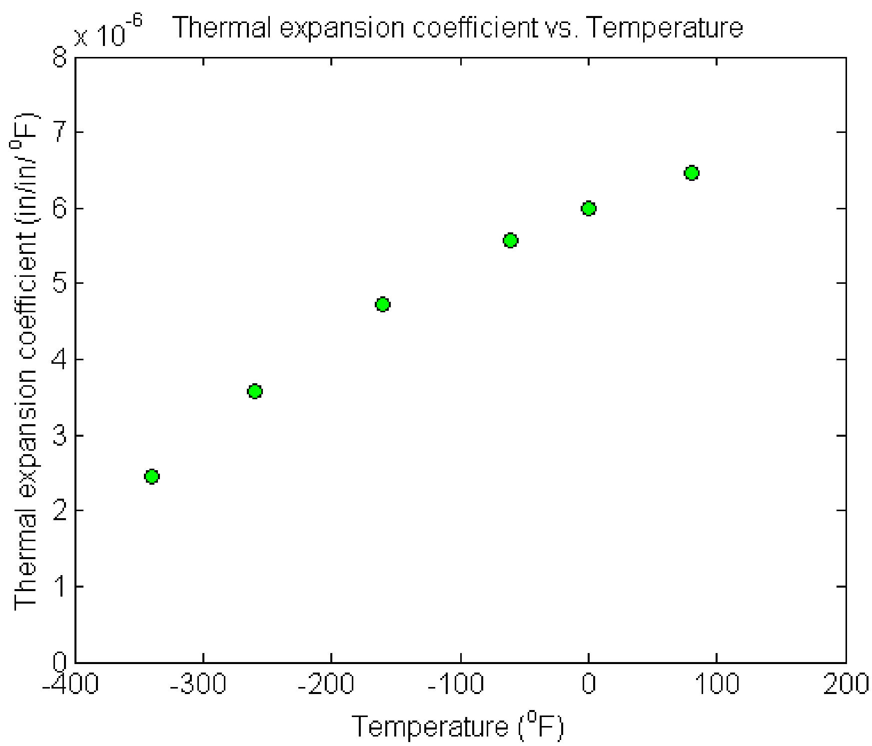

Figure 1 Thermal expansion coefficient vs. temperature.

Determine the value of the coefficient of thermal expansion at $T = -14\,°\text{F}$ using Newton's divided difference method of interpolation and a first order polynomial.

Solution

For linear interpolation, the coefficient of thermal expansion is given by

$$\alpha(T) = b_0 + b_1(T - T_0)$$

Since we want to find the coefficient of thermal expansion at $T = -14$ and we are using a first order polynomial, we need to choose the two data points that are closest to $T = -14$ that also bracket $T = -14$ to evaluate it. The two points are $T_0 = 0$ and $T_1 = -60$.

Then

$$T_0 = 0, \quad \alpha(T_0) = 6.00 \times 10^{-6}$$

$$T_1 = -60, \quad \alpha(T_1) = 5.58 \times 10^{-6}$$

gives

$$\begin{aligned} b_0 &= \alpha(T_0) \\ &= 6.00 \times 10^{-6} \end{aligned}$$

$$\begin{aligned} b_1 &= \frac{\alpha(T_1) - \alpha(T_0)}{T_1 - T_0} \\ &= \frac{5.58 \times 10^{-6} - 6.00 \times 10^{-6}}{-60 - 0} \\ &= 0.007 \times 10^{-6} \end{aligned}$$

Hence

$$\alpha(T) = b_0 + b_1(T - T_0)$$
$$= 6.00\times10^{-6} + 0.007\times10^{-6}(T - 0), \qquad -60 \le T \le 0$$

At $T = -14$

$$\alpha(-14) = 6.00\times10^{-6} + 0.007\times10^{-6}(-14 - 0)$$
$$= 5.902\times10^{-6} \text{ in/in/°F}$$

If we expand

$$\alpha(T) = 6.00\times10^{-6} + 0.007\times10^{-6}(T - 0), \qquad -60 \le T \le 0$$

we get

$$\alpha(T) = 6.00\times10^{-6} + 0.007\times10^{-6}T, \qquad -60 \le T \le 0$$

This is the same expression that was obtained with the direct method.

Example 2

For the purpose of shrinking a trunnion into a hub, the reduction of diameter ΔD of a trunnion shaft by cooling it through a temperature change of ΔT is given by

$$\Delta D = D\alpha\Delta T$$

where

$D =$ original diameter (in.)

$\alpha =$ coefficient of thermal expansion at average temperature (in/in/°F)

The trunnion is cooled from $80°\text{F}$ to $-108°\text{F}$, giving the average temperature as $-14°\text{F}$. The table of the coefficient of thermal expansion vs. temperature data is given in Table 2.

Table 2 Thermal expansion coefficient as a function of temperature.

Temperature, T (°F)	Thermal Expansion Coefficient, α (in/in/°F)
80	6.47×10^{-6}
0	6.00×10^{-6}
–60	5.58×10^{-6}
–160	4.72×10^{-6}
–260	3.58×10^{-6}
–340	2.45×10^{-6}

Determine the value of the coefficient of thermal expansion at $T = -14°\text{F}$ using Newton's divided difference method of interpolation and a second order polynomial. Find the absolute relative approximate error for the second order polynomial approximation.

Solution

For quadratic interpolation, the coefficient of thermal expansion is given by

$$\alpha(T) = b_0 + b_1(T - T_0) + b_2(T - T_0)(T - T_1)$$

Since we want to find the coefficient of thermal expansion at $T = -14,$ we need to choose the three data points that are closest to $T = -14$ that also bracket $T = -14$ to evaluate it. The three points are $T_0 = 80,\ T_1 = 0$ and $T_2 = -60$.

Then

$$T_0 = 80, \quad \alpha(T_0) = 6.47\times 10^{-6}$$
$$T_1 = 0, \quad \alpha(T_1) = 6.00\times 10^{-6}$$
$$T_2 = -60, \quad \alpha(T_2) = 5.58\times 10^{-6}$$

gives

$$\begin{aligned} b_0 &= \alpha(T_0) \\ &= 6.47\times 10^{-6} \end{aligned}$$

$$\begin{aligned} b_1 &= \frac{\alpha(T_1)-\alpha(T_0)}{T_1 - T_0} \\ &= \frac{6.00\times 10^{-6} - 6.47\times 10^{-6}}{0-80} \\ &= 5.875\times 10^{-9} \end{aligned}$$

$$\begin{aligned} b_2 &= \frac{\dfrac{\alpha(T_2)-\alpha(T_1)}{T_2 - T_1} - \dfrac{\alpha(T_1)-\alpha(T_0)}{T_1 - T_0}}{T_2 - T_0} \\ &= \frac{\dfrac{5.58\times 10^{-6} - 6.00\times 10^{-6}}{-60-0} - \dfrac{6.00\times 10^{-6} - 6.47\times 10^{-6}}{0-80}}{-60-80} \\ &= \frac{0.007\times 10^{-6} - 0.005875\times 10^{-6}}{-140} \\ &= -8.0357\times 10^{-12} \end{aligned}$$

Hence

$$\begin{aligned} \alpha(T) &= b_0 + b_1(T-T_0) + b_2(T-T_0)(T-T_1) \\ &= 6.47\times 10^{-6} + 5.875\times 10^{-9}(T-80) - 8.0357\times 10^{-12}(T-80)(T-0),\ -60 \le T \le 80 \end{aligned}$$

At $T = -14,$

$$\begin{aligned} \alpha(-14) &= 6.47\times 10^{-6} + 5.875\times 10^{-9}(-14-80) - 8.0357\times 10^{-12}(-14-80)(-14-0) \\ &= 5.9072\times 10^{-6}\ \text{in/in/°F} \end{aligned}$$

The absolute relative approximate error $|\in_a|$ obtained between the results from the first and second order polynomial is

$$\begin{aligned} |\in_a| &= \left|\frac{5.9072\times 10^{-6} - 5.902\times 10^{-6}}{5.9072\times 10^{-6}}\right|\times 100 \\ &= 0.087605\% \end{aligned}$$

If we expand

$$\alpha(T) = 6.47\times 10^{-6} + 5.875\times 10^{-9}(T-80) - 8.0357\times 10^{-12}(T-80)(T-0),\ -60 \le T \le 80$$

we get

$$\alpha(T) = 6.00\times10^{-6} + 6.5179\times10^{-9}T - 8.0357\times10^{-12}T^2, \quad -60 \le T \le 80$$

This is the same expression that was obtained with the direct method.

Example 3

For the purpose of shrinking a trunnion into a hub, the reduction of diameter ΔD of a trunnion shaft by cooling it through a temperature change of ΔT is given by

$$\Delta D = D\alpha\Delta T$$

where

$D =$ original diameter (in.)

$\alpha =$ coefficient of thermal expansion at average temperature (in/in/°F)

The trunnion is cooled from 80°F to −108°F, giving the average temperature as −14°F. The table of the coefficient of thermal expansion vs. temperature data is given in Table 3.

Table 3 Thermal expansion coefficient as a function of temperature.

Temperature, T (°F)	Thermal Expansion Coefficient, α (in/in/°F)
80	6.47×10^{-6}
0	6.00×10^{-6}
–60	5.58×10^{-6}
–160	4.72×10^{-6}
–260	3.58×10^{-6}
–340	2.45×10^{-6}

a) Determine the value of the coefficient of thermal expansion at $T = -14$°F using Newton's divided difference method of interpolation and a third order polynomial. Find the absolute relative approximate error for the third order polynomial approximation.

b) The actual reduction in diameter is given by

$$\Delta D = D\int_{T_r}^{T_f} \alpha dT$$

where

$T_r =$ room temperature (°F)

$T_f =$ temperature of cooling medium (°F)

Since

$T_r = 80$°F

$T_f = -108$°F

$$\Delta D = D\int_{80}^{-108} \alpha dT$$

Find out the percentage difference in the reduction in the diameter by the above integral formula and the result using the thermal expansion coefficient from part (a).

Solution

a) For cubic interpolation, the coefficient of thermal expansion is given by

$$\alpha(T) = b_0 + b_1(T - T_0) + b_2(T - T_0)(T - T_1) + b_3(T - T_0)(T - T_1)(T - T_2)$$

Since we want to find the coefficient of thermal expansion at $T = -14$ and we are using a third order polynomial, we need to choose the four data points that are closest to $T = -14$ and bracket $T = -14$. These four data points are $T_0 = 80,\ T_1 = 0,\ T_2 = -60$ and $T_3 = -160.$

Then

$$T_0 = 80, \quad \alpha(T_0) = 6.47 \times 10^{-6}$$

$$T_1 = 0, \quad \alpha(T_1) = 6.00 \times 10^{-6}$$

$$T_2 = -60, \quad \alpha(T_2) = 5.58 \times 10^{-6}$$

$$T_3 = -160, \quad \alpha(\mathrm{T}_3) = 4.72 \times 10^{-6}$$

gives

$$b_0 = \alpha[T_0]$$

$$= \alpha(T_0)$$

$$= 6.47 \times 10^{-6}$$

$$b_1 = \alpha[T_1, T_0]$$

$$= \frac{\alpha(T_1) - \alpha(T_0)}{T_1 - T_0}$$

$$= \frac{6.00 \times 10^{-6} - 6.47 \times 10^{-6}}{0 - 80}$$

$$= 5.875 \times 10^{-9}$$

$$b_2 = \alpha[T_2, T_1, T_0]$$

$$= \frac{\alpha[T_2, T_1] - \alpha[T_1, T_0]}{T_2 - T_0}$$

$$\alpha[T_2, T_1] = \frac{\alpha(T_2) - \alpha(T_1)}{T_2 - T_1}$$

$$= \frac{5.58 \times 10^{-6} - 6.00 \times 10^{-6}}{-60 - 0}$$

$$= 0.007 \times 10^{-6}$$

$$\alpha[T_1, T_0] = 5.875 \times 10^{-9}$$

$$b_2 = \frac{\alpha[T_2, T_1] - \alpha[T_1, T_0]}{T_2 - T_0}$$

$$= \frac{0.007 \times 10^{-6} - 0.005875 \times 10^{-6}}{-60 - 80}$$

$$= -8.0357 \times 10^{-12}$$

$$b_3 = \alpha[T_3, T_2, T_1, T_0]$$
$$= \frac{\alpha[T_3, T_2, T_1] - \alpha[T_2, T_1, T_0]}{T_3 - T_0}$$
$$\alpha[T_3, T_2, T_1] = \frac{\alpha[T_3, T_2] - \alpha[T_2, T_1]}{T_3 - T_1}$$
$$\alpha[T_3, T_2] = \frac{\alpha(T_3) - \alpha(T_2)}{T_3 - T_2}$$
$$= \frac{4.72\times10^{-6} - 5.58\times10^{-6}}{-160 + 60}$$
$$= 0.0086\times10^{-6}$$
$$\alpha[T_2, T_1] = 0.007\times10^{-6}$$
$$\alpha[T_3, T_2, T_1] = \frac{\alpha[T_3, T_2] - \alpha[T_2, T_1]}{T_3 - T_1}$$
$$= \frac{0.0086\times10^{-6} - 0.007\times10^{-6}}{-160 - 0}$$
$$= -10^{-11}$$
$$\alpha[T_2, T_1, T_0] = -8.0357\times10^{-12}$$
$$b_3 = \alpha[T_3, T_2, T_1, T_0]$$
$$= \frac{\alpha[T_3, T_2, T_1] - \alpha[T_2, T_1, T_0]}{T_3 - T_0}$$
$$= \frac{-10^{-11} + 8.0357\times10^{-12}}{-160 - 80}$$
$$= 8.1845\times10^{-15}$$

Hence

$$\alpha(T) = b_0 + b_1(T - T_0) + b_2(T - T_0)(T - T_1) + b_3(T - T_0)(T - T_1)(T - T_2)$$
$$= 6.47\times10^{-6} + 5.875\times10^{-9}(T - 80) - 8.0357\times10^{-12}(T - 80)(T - 0)$$
$$+ 8.1845\times10^{-15}(T - 80)(T - 0)(T + 60)$$

At $T = -14$,

$$\alpha(-14) = 6.47\times10^{-6} + 5.875\times10^{-9}(-14 - 80) - 8.0357\times10^{-12}(-14 - 80)(-14 - 0)$$
$$+ 8.1845\times10^{-15}(-14 - 80)(-14 - 0)(-14 + 60)$$
$$= 5.9077\times 10^{-6}\ \text{in/in/°F}$$

The absolute relative approximate error $|\in_a|$ obtained between the results from the second and third order polynomial is

$$|\in_a| = \left|\frac{5.9077\times10^{-6} - 5.9072\times10^{-6}}{5.9077\times10^{-6}}\right|\times100$$
$$= 0.0083867\%$$

b) In finding the percentage difference in the reduction in diameter, we can rearrange the integral formula to

$$\frac{\Delta D}{D} = \int_{T_r}^{T_f} \alpha dT$$

and since we know from part (a) that

$$\alpha(T) = 6.47\times10^{-6} + 5.875\times10^{-9}(T-80) - 8.0357\times10^{-12}(T-80)(T-0)$$
$$+ 8.1845\times10^{-15}(T-80)(T-0)(T+60), \; -160 \le T \le 80$$

Combining like terms, we get

$$\alpha(T) = 6.00\times10^{-6} + 6.4786\times10^{-9}T - 8.1994\times10^{-12}T^2 + 8.1845\times10^{-15}T^3, \; -160 \le T \le 80$$

We see that we can use the integral formula in the range from $T_f = -108\,°\text{F}$ to $T_r = 80\,°\text{F}$

Therefore,

$$\frac{\Delta D}{D} = \int_{T_r}^{T_f} \alpha dT$$
$$= \int_{80}^{-108} (6.00\times10^{-6} + 6.4786\times10^{-3}T - 8.1994\times10^{-6}T^2 + 8.1845\times10^{-9}T^3)dT$$
$$= \left[6.00\times10^{-6}T + 6.4786\times10^{-9}\frac{T^2}{2} - 8.1994\times10^{-9}\frac{T^3}{3} + 8.1845\times10^{-15}\frac{T^4}{4} \right]_{80}^{-108}$$
$$= -1105.9\times10^{-6}$$

So $\frac{\Delta D}{D} = -1105.9\times10^{-6}$ in/in using the actual reduction in diameter integral formula. If we use the average value for the coefficient of thermal expansion from part (a), we get

$$\frac{\Delta D}{D} = \alpha\Delta T$$
$$= \alpha(T_f - T_r)$$
$$= 5.9077\times10^{-6}(-108-80)$$
$$= -1110.6\times10^{-6}$$

and $\frac{\Delta D}{D} = -1110.6\times10^{-6}$ in/in using the average value of the coefficient of thermal expansion using a third order polynomial. Considering the integral to be the more accurate calculation, the percentage difference would be

$$|\in_a| = \left| \frac{-1105.9\times10^{-6} + 1110.6\times10^{-6}}{-1105.9\times10^{-6}} \right| \times 100$$
$$= 0.42775\%$$

INTERPOLATION	
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