

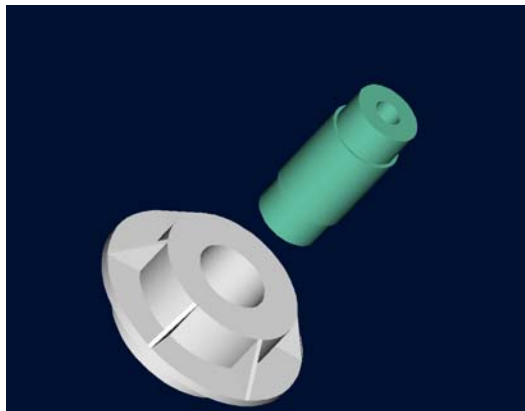
## Chapter 03.03

### Bisection Method of Solving a Nonlinear Equation- More Examples

#### Mechanical Engineering

##### Example 1

A trunnion has to be cooled before it is shrink fitted into a steel hub.



**Figure 1** Trunnion to be slid through the hub after contracting.

The equation that gives the temperature  $T_f$  to which the trunnion has to be cooled to obtain the desired contraction is given by

$$f(T_f) = -0.50598 \times 10^{-10} T_f^3 + 0.38292 \times 10^{-7} T_f^2 + 0.74363 \times 10^{-4} T_f + 0.88318 \times 10^{-2} = 0$$

Use the bisection method of finding roots of equations to find the temperature  $T_f$  to which the trunnion has to be cooled. Conduct three iterations to estimate the root of the above equation. Find the absolute relative approximate error at the end of each iteration and the number of significant digits at least correct at the end of each iteration.

##### Solution

From the designer's records for the previous bridge, the temperature to which the trunnion was cooled was  $-108^\circ\text{F}$ . Hence assuming the temperature to be between  $-100^\circ\text{F}$  and  $-150^\circ\text{F}$ , we have

$$T_{f,\ell} = -150^\circ\text{F}, T_{f,u} = -100^\circ\text{F}$$

Check if the function changes sign between  $T_{f,\ell}$  and  $T_{f,u}$ .

$$\begin{aligned}
 f(T_{f,\ell}) &= f(-150) \\
 &= -0.50598 \times 10^{-10} (-150)^3 + 0.38292 \times 10^{-7} (-150)^2 \\
 &\quad + 0.74363 \times 10^{-4} (-150) + 0.88318 \times 10^{-2} \\
 &= -1.2903 \times 10^{-3} \\
 f(T_{f,u}) &= f(-100) \\
 &= -0.50598 \times 10^{-10} (-100)^3 + 0.38292 \times 10^{-7} (-100)^2 \\
 &\quad + 0.74363 \times 10^{-4} (-100) + 0.88318 \times 10^{-2} \\
 &= 1.8290 \times 10^{-3}
 \end{aligned}$$

Hence

$$f(T_{f,\ell})f(T_{f,u}) = f(-150)f(-100) = (-1.2903 \times 10^{-3})(1.8290 \times 10^{-3}) < 0$$

So there is at least one root between  $T_{f,\ell}$  and  $T_{f,u}$  that is between  $-150$  and  $-100$ .

### Iteration 1

The estimate of the root is

$$\begin{aligned}
 T_{f,m} &= \frac{T_{f,\ell} + T_{f,u}}{2} \\
 &= \frac{-150 + (-100)}{2} \\
 &= -125
 \end{aligned}$$

$$\begin{aligned}
 f(T_{f,m}) &= f(-125) \\
 &= -0.50598 \times 10^{-10} (-125)^3 + 0.38292 \times 10^{-7} (-125)^2 \\
 &\quad + 0.74363 \times 10^{-4} (-125) + 0.88318 \times 10^{-2} \\
 &= 2.3356 \times 10^{-4}
 \end{aligned}$$

$$f(T_{f,\ell})f(T_{f,m}) = f(-150)f(-125) = (-1.2903 \times 10^{-3})(2.3356 \times 10^{-4}) < 0$$

Hence the root is bracketed between  $T_{f,\ell}$  and  $T_{f,m}$ , that is, between  $-150$  and  $-125$ .

So, the lower and upper limits of the new bracket are

$$T_{f,\ell} = -150, T_{f,u} = -125$$

At this point, the absolute relative approximate error  $|\epsilon_a|$  cannot be calculated, as we do not have a previous approximation.

### Iteration 2

The estimate of the root is

$$\begin{aligned}
 T_{f,m} &= \frac{T_{f,\ell} + T_{f,u}}{2} \\
 &= \frac{-150 + (-125)}{2} \\
 &= -137.5
 \end{aligned}$$

$$\begin{aligned}
 f(T_{f,m}) &= f(-137.5) \\
 &= -0.50598 \times 10^{-10} (-137.5)^3 + 0.38292 \times 10^{-7} (-137.5)^2 \\
 &\quad + 0.74363 \times 10^{-4} (-137.5) + 0.88318 \times 10^{-2} \\
 &= -5.3762 \times 10^{-4} \\
 f(T_{f,m})f(T_{f,u}) &= f(-137.5)f(-125) = (-5.3762 \times 10^{-4})(2.3356 \times 10^{-4}) < 0
 \end{aligned}$$

Hence, the root is bracketed between  $T_{f,m}$  and  $T_{f,u}$ , that is, between  $-125$  and  $-137.5$ .

So the lower and upper limits of the new bracket are

$$T_{f,\ell} = -137.5, T_{f,u} = -125$$

The absolute relative approximate error  $|\epsilon_a|$  at the end of Iteration 2 is

$$\begin{aligned}
 |\epsilon_a| &= \left| \frac{T_{f,m}^{\text{new}} - T_{f,m}^{\text{old}}}{T_{f,m}^{\text{new}}} \right| \times 100 \\
 &= \left| \frac{-137.5 - (-125)}{-137.5} \right| \times 100 \\
 &= 9.0909\%
 \end{aligned}$$

None of the significant digits are at least correct in the estimated root of

$$T_{f,m} = -137.5$$

as the absolute relative approximate error is greater than 5%.

### Iteration 3

The estimate of the root is

$$\begin{aligned}
 T_{f,m} &= \frac{T_{f,\ell} + T_{f,u}}{2} \\
 &= \frac{-137.5 + (-125)}{2} \\
 &= -131.25
 \end{aligned}$$

$$\begin{aligned}
 f(T_{f,m}) &= f(-131.25) \\
 &= -0.50598 \times 10^{-10} (-131.25)^3 + 0.38292 \times 10^{-7} (-131.25)^2 \\
 &\quad + 0.74363 \times 10^{-4} (-131.25) + 0.88318 \times 10^{-2} \\
 &= -1.54303 \times 10^{-4}
 \end{aligned}$$

$$f(T_{f,\ell})f(T_{f,m}) = f(-125)f(-131.25) = (2.3356 \times 10^{-4})(-1.5430 \times 10^{-4}) < 0$$

Hence, the root is bracketed between  $T_{f,\ell}$  and  $T_{f,m}$ , that is, between  $-125$  and  $-131.25$ .

So the lower and upper limits of the new bracket are

$$T_{f,\ell} = -131.25, T_{f,u} = -125$$

The absolute relative approximate error  $|\epsilon_a|$  at the ends of Iteration 3 is

$$|\epsilon_a| = \left| \frac{T_{f,m}^{\text{new}} - T_{f,m}^{\text{old}}}{T_{f,m}^{\text{new}}} \right| \times 100$$

$$= \left| \frac{-131.25 - (-137.5)}{-131.25} \right| \times 100$$

$$= 4.7619\%$$

The number of significant digits at least correct is 1.

Seven more iterations were conducted and these iterations are shown in the Table 1 below.

**Table 1** Root of  $f(x) = 0$  as function of number of iterations for bisection method.

Iteration	$T_{f,\ell}$	$T_{f,u}$	$T_{f,m}$	$ \epsilon_a  \%$	$f(T_{f,m})$
1	-150	-100	-125	-----	$2.3356 \times 10^{-4}$
2	-150	-125	-137.5	9.0909	$-5.3762 \times 10^{-4}$
3	-137.5	-125	-131.25	4.7619	$-1.5430 \times 10^{-4}$
4	-131.25	-125	-128.13	2.4390	$3.9065 \times 10^{-5}$
5	-131.25	-128.13	-129.69	1.2048	$-5.7760 \times 10^{-5}$
6	-129.69	-123.13	-128.91	0.60606	$-9.3826 \times 10^{-6}$
7	-128.91	-123.13	-128.52	0.30395	$1.4838 \times 10^{-5}$
8	-128.91	-128.52	-128.71	0.15175	$2.7228 \times 10^{-6}$
9	-128.91	-128.71	-128.81	0.075815	$-3.3305 \times 10^{-6}$
10	-128.81	-128.71	-128.76	0.037922	$-3.0396 \times 10^{-7}$

At the end of the 10<sup>th</sup> iteration,

$$|\epsilon_a| = 0.037922\%$$

Hence, the number of significant digits at least correct is given by the largest value of  $m$  for which

$$|\epsilon_a| \leq 0.5 \times 10^{2-m}$$

$$0.037922 \leq 0.5 \times 10^{2-m}$$

$$0.075844 \leq 10^{2-m}$$

$$\log(0.075844) \leq 2 - m$$

$$m \leq 2 - \log(0.075844) = 3.1201$$

So

$$m = 3$$

The number of significant digits at least correct in the estimated root of  $-128.76$  is 3.

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#### NONLINEAR EQUATIONS

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Topic      Bisection Method-More Examples

Summary    Examples of Bisection Method

Major        Mechanical Engineering

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