

## Chapter 07.06

### Gauss Quadrature Rule for Integration-More Examples

### Industrial Engineering

#### Example 1

A company advertises that every roll of toilet paper has at least 250 sheets. The probability that there are 250 or more sheets in the toilet paper is given by

$$P(y \geq 250) = \int_{250}^{\infty} 0.3515 e^{-0.3881(y-252.2)^2} dy$$

Approximating the above integral as

$$P(y \geq 250) = \int_{250}^{270} 0.3515 e^{-0.3881(y-252.2)^2} dy$$

- Use two-point Gauss Quadrature Rule to find the probability.
- Find the absolute relative true error.

#### Solution

a) First, change the limits of integration from  $[250, 270]$  to  $[-1, 1]$  using

$$a = 250$$

$$b = 270$$

$$\int_a^b f(y) dy = \frac{b-a}{2} \int_{-1}^1 f\left(\frac{b-a}{2}y + \frac{b+a}{2}\right) dy$$

gives

$$\begin{aligned} \int_{250}^{270} f(y) dy &= \frac{270-250}{2} \int_{-1}^1 f\left(\frac{270-250}{2}y + \frac{270+250}{2}\right) dy \\ &= 10 \int_{-1}^1 f(10y + 260) dy \end{aligned}$$

Next, get weighting factors and function argument values for the two point rule,

$$c_1 = 1.0000$$

$$y_1 = -0.57735$$

$$c_2 = 1.0000$$

$$y_2 = 0.57735$$

Now we can use the Gauss Quadrature formula

$$10 \int_{-1}^1 f(10y + 260) dy \approx 10 [c_1 f(10y_1 + 260) + c_2 f(10y_2 + 260)]$$

$$\begin{aligned}
&\approx 10[f(10(-0.57735) + 260) + f(10(0.57735) + 260)] \\
&\approx 10[f(254.23) + f(265.77)] \\
&\approx 10[(0.071407) + (3.1070 \times 10^{-32})] \\
&\approx 0.71408
\end{aligned}$$

since

$$\begin{aligned}
f(254.23) &= 0.3515e^{-0.3881(254.23-252.2)^2} \\
&= 0.071407 \\
f(265.77) &= 0.3515e^{-0.3881(265.77-252.2)^2} \\
&= 3.1070 \times 10^{-32}
\end{aligned}$$

b) The absolute relative true error,  $|\epsilon_t|$ , is (Exact value = 0.97377)

$$\begin{aligned}
|\epsilon_t| &= \left| \frac{\text{True Value} - \text{Approximate Value}}{\text{True Value}} \right| \times 100\% \\
|\epsilon_t| &= \left| \frac{0.97377 - 0.71015}{0.97377} \right| \times 100\% \\
&= 26.669\%
\end{aligned}$$

### Example 2

A company advertises that every roll of toilet paper has at least 250 sheets. The probability that there are 250 or more sheets in the toilet paper is given by

$$P(y \geq 250) = \int_{250}^{\infty} 0.3515 e^{-0.3881(y-252.2)^2} dy$$

Approximating the above integral as

$$P(y \geq 250) = \int_{250}^{270} 0.3515 e^{-0.3881(y-252.2)^2} dy$$

- Use three-point Gauss Quadrature to find the probability.
- Find the absolute relative true error.

### Solution

a) First, change the limits of integration from  $[250, 270]$  to  $[-1, 1]$  using

$$a = 250$$

$$b = 270$$

$$\int_a^b f(y) dy = \frac{b-a}{2} \int_{-1}^1 f\left(\frac{b-a}{2}y + \frac{b+a}{2}\right) dy$$

gives

$$\begin{aligned}
\int_{250}^{270} f(y) dy &= \frac{270-250}{2} \int_{-1}^1 f\left(\frac{270-250}{2}y + \frac{270+250}{2}\right) dy \\
&= 10 \int_{-1}^1 f(10y + 260) dy
\end{aligned}$$

The weighting factors and function argument values are

$$c_1 = 0.55556$$

$$x_1 = -0.77460$$

$$c_2 = 0.88889$$

$$x_2 = 0.0000$$

$$c_3 = 0.55556$$

$$x_3 = 0.77460$$

and the formula is

$$\begin{aligned} 10 \int_{-1}^1 f(10y + 260) dy &\approx 10 [c_1 f(10y_1 + 260) + c_2 f(10y_2 + 260) + c_3 f(10y_3 + 260)] \\ &\approx 10 \left[ \begin{array}{l} 0.55556 f(10(-0.77460) + 260) \\ + 0.88889 f(10(0.0000) + 260) \\ + 0.55556 f(10(0.77460) + 260) \end{array} \right] \\ &\approx 10 \left[ \begin{array}{l} 0.55556 f(252.25) + 0.88889 f(260.00) \\ + 0.55556 f(267.75) \end{array} \right] \\ &\approx 10 \left[ \begin{array}{l} 0.55556(0.35110) + 0.88889(1.9560 \times 10^{-11}) \\ + 0.55556(6.4763 \times 10^{-42}) \end{array} \right] \\ &\approx 1.9509 \end{aligned}$$

since

$$\begin{aligned} f(252.25) &= 0.3515 e^{-0.3881(252.25-252.2)^2} \\ &= 0.35110 \end{aligned}$$

$$\begin{aligned} f(260.00) &= 0.3515 e^{-0.3881(260.00-252.2)^2} \\ &= 1.9560 \times 10^{-11} \end{aligned}$$

$$\begin{aligned} f(267.75) &= 0.3515 e^{-0.3881(267.75-252.2)^2} \\ &= 6.4763 \times 10^{-42} \end{aligned}$$

b) The absolute relative true error,  $|\epsilon_t|$ , is (exact value = 0.97377)

$$\begin{aligned} |\epsilon_t| &= \left| \frac{\text{True Value} - \text{Approximate Value}}{\text{True Value}} \right| \times 100 \% \\ |\epsilon_t| &= \left| \frac{0.97377 - 1.9509}{0.97377} \right| \times 100 \% \\ &= 100.31 \% \end{aligned}$$