Chapter 06.03
Linear Regression–More Examples
Industrial Engineering

Example 1
As machines are used over long periods of time, the output product can get off target. Below is the average value of how much off target a product is getting manufactured as a function of machine use.

Table 1 Off target value as a function of machine use.

<table>
<thead>
<tr>
<th>Hours of Machine Use, ( t )</th>
<th>30</th>
<th>33</th>
<th>34</th>
<th>35</th>
<th>39</th>
<th>44</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Millimeters Off Target, ( h )</td>
<td>1.10</td>
<td>1.21</td>
<td>1.25</td>
<td>1.23</td>
<td>1.30</td>
<td>1.40</td>
<td>1.42</td>
</tr>
</tbody>
</table>

Regress the data to \( h = a_0 + a_1 t \). Find when the product will be 2 mm off target.

Solution
Table 2 shows the summations needed for the calculation of the constants of the regression model.

Table 2 Tabulation of data for calculation of needed summations.

<table>
<thead>
<tr>
<th>( i )</th>
<th>( t )</th>
<th>( h )</th>
<th>( t^2 )</th>
<th>( t \times h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sum_{i=1}^7 )</td>
<td>260</td>
<td>8.91</td>
<td>9852</td>
<td>334.68</td>
</tr>
</tbody>
</table>

\( I \) | \( t \) Hours | \( h \) Millimeters | \( t^2 \) | \( t \times h \) Millimeter-Hour
\hline
\(-\) | \( - \) | \( - \) | \( - \) | \( - \) | \\
1   | 30   | 1.10  | 900   | 33    |
2   | 33   | 1.21  | 1089  | 39.93 |
3   | 34   | 1.25  | 1156  | 42.50 |
4   | 35   | 1.23  | 1225  | 43.05 |
5   | 39   | 1.30  | 1521  | 50.70 |
6   | 44   | 1.40  | 1936  | 61.6  |
7   | 45   | 1.42  | 2025  | 63.9  |
\hline
\[ n = 7 \]
\[ a_i = \frac{n \sum t_i h_i - \sum t_i \sum h_i}{n \sum t_i^2 - \left( \sum t_i \right)^2} \]
\[ = \frac{7(334.68) - (260)(8.91)}{7(9852) - (260)^2} \]
\[ = 0.019179 \text{ mm-h} \]

\[ \bar{h} = \frac{\sum h_i}{n} \]
\[ = \frac{8.91}{7} \]
\[ = 1.2729 \text{ mm} \]

\[ \bar{t} = \frac{\sum t_i}{n} \]
\[ = \frac{260}{7} \]
\[ = 37.143 \text{ h} \]

\[ a_0 = \bar{h} - a_i \bar{t} \]
\[ = 1.2729 - (0.019179)(37.143) \]
\[ = 0.56050 \text{ mm-h} \]
\[ h = 0.56050 + 0.019179t \]

**Figure 1** Linear regression of hours of use vs. millimeters off target.

Solving for \( h = 2 \) mm, the regression model is \( h = 0.56050 + 0.019179t \)

\[
2 = 0.56050 + 0.019179t \\
t = \frac{2 - 0.56050}{0.019179} \\
t = 75.056 \text{ hours}
\]