

## Chapter 04.06

### Gaussian Elimination – More Examples

#### Industrial Engineering

##### Example 1

To find the number of toys a company should manufacture per day to optimally use their injection-molding machine and the assembly line, one needs to solve the following set of equations. The unknowns are the number of toys for boys,  $x_1$ , the number of toys for girls,  $x_2$ , and the number of unisexual toys,  $x_3$ .

$$\begin{bmatrix} 0.3333 & 0.1667 & 0.6667 \\ 0.1667 & 0.6667 & 0.3333 \\ 1.05 & -1.00 & 0.00 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 756 \\ 1260 \\ 0 \end{bmatrix}$$

Find the values of  $x_1$ ,  $x_2$ , and  $x_3$  using naïve Gauss elimination.

##### Solution

##### Forward Elimination of Unknowns

Since there are three equations, there will be two steps of forward elimination of unknowns.

##### First step

Divide Row 1 by 0.3333 and then multiply it by 0.1667, that is, multiply Row 1 by  $0.1667/0.3333 = 0.50015$ .

$$\text{Row 1} \times (0.50015) = [0.1667 \quad 0.083375 \quad 0.33345] \quad [378.11]$$

Subtract the result from Row 2 to get

$$\begin{bmatrix} 0.3333 & 0.1667 & 0.6667 \\ 0 & 0.58332 & -0.00015002 \\ 1.05 & -1.00 & 0.00 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 756 \\ 881.89 \\ 0 \end{bmatrix}$$

Divide Row 1 by 0.3333 and then multiply it by 1.05, that is, multiply Row 1 by  $1.05/0.3333 = 3.1503$ .

$$\text{Row 1} \times (3.1503) = [1.05 \quad 0.52516 \quad 2.1003] \quad [2381.6]$$

Subtract the result from Row 3 to get

$$\begin{bmatrix} 0.3333 & 0.1667 & 0.6667 \\ 0 & 0.58332 & -0.00015002 \\ 0 & -1.5252 & -2.1007 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 756 \\ 881.89 \\ -2381.6 \end{bmatrix}$$

**Second step**

We now divide Row 2 by 0.58332 and then multiply it by  $-1.5252$ , that is, multiply Row 2 by  $-1.5252/0.58332 = -2.6146$ .

$$\text{Row } 2 \times (-2.6146) = \begin{bmatrix} 0 & -1.5252 & 3.9223 \times 10^{-4} \end{bmatrix} \quad [-2305.8]$$

Subtract the result from Row 3 we get

$$\begin{bmatrix} 0.3333 & 0.1667 & 0.6667 \\ 0 & 0.58332 & -0.00015002 \\ 0 & 0 & -2.1007 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 756 \\ 881.89 \\ -75.864 \end{bmatrix}$$

**Back Substitution**

From the third equation,

$$-2.1008x_3 = -75.864$$

$$\begin{aligned} x_3 &= \frac{-75.864}{-2.1007} \\ &= 36.113 \end{aligned}$$

Substituting the value of  $x_3$  in the second equation,

$$0.58332x_2 + (-0.00015002)x_3 = 881.89$$

$$\begin{aligned} x_2 &= \frac{881.89 - (-0.00015002)x_3}{0.58332} \\ &= \frac{881.89 - (-0.00015002) \times 36.113}{0.58332} \\ &= 1511.8 \end{aligned}$$

Substituting the values of  $x_2$  and  $x_3$  in the first equation,

$$0.3333x_1 + 0.1667x_2 + 0.6667x_3 = 756$$

$$\begin{aligned} x_1 &= \frac{756 - 0.1667x_2 - 0.6667x_3}{0.3333} \\ &= \frac{756 - 0.1667 \times 1511.8 - 0.6667 \times 36.113}{0.3333} \\ &= 1439.8 \end{aligned}$$

Hence the solution vector is

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1439.8 \\ 1511.8 \\ 36.113 \end{bmatrix}$$

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**SIMULTANEOUS LINEAR EQUATIONS**

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Topic	Gaussian Elimination – More Examples
Summary	Examples of Gaussian elimination
Major	Industrial Engineering
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