

## Chapter 06.04

# Nonlinear Models for Regression-More Examples

## Electrical Engineering

### Example 1

To be able to draw road networks from aerial images, light intensities are measured at different pixel locations. The following intensities are given as a function of pixel location.

**Table 1** Light intensities as a function of pixel location.

Pixel Location, $k$	Intensity, $y$
-3	119
-2	165
-1	231
0	243
1	244
2	214
3	136

Regress the above data to a second order polynomial

$$y = a_0 + a_1k + a_2k^2$$

### Solution

Table 2 shows the summations needed for the calculation of the constants of the regression model.

**Table 2** Summations for calculating constants of model.

$i$	Pixel Location, $k$	Intensity, $y$	$k^2$	$k^3$	$k^4$	$k \times y$	$k^2 \times y$
1	-3	119	9	-27	81	-357	1071
2	-2	165	4	-8	16	-330	660
3	-1	231	1	-1	1	-231	231
4	0	243	0	0	0	0	0
5	1	244	1	1	1	244	244
6	2	214	4	8	16	428	856
7	3	136	9	27	81	408	1224
$\sum_{i=1}^7$	0	1352	28	0	196	162	4286

$y = a_0 + a_1k + a_2k^2$  is the quadratic relationship between the pixel location and intensity where the coefficients  $a_0$ ,  $a_1$ ,  $a_2$  are found as follows

$$\begin{bmatrix} n & \left(\sum_{i=1}^n k_i\right) & \left(\sum_{i=1}^n k_i^2\right) \\ \left(\sum_{i=1}^n k_i\right) & \left(\sum_{i=1}^n k_i^2\right) & \left(\sum_{i=1}^n k_i^3\right) \\ \left(\sum_{i=1}^n k_i^2\right) & \left(\sum_{i=1}^n k_i^3\right) & \left(\sum_{i=1}^n k_i^4\right) \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n k_i y_i \\ \sum_{i=1}^n k_i^2 y_i \end{bmatrix}$$

$$n = 7$$

$$\sum_{i=1}^7 k_i = 0$$

$$\sum_{i=1}^7 k_i^2 = 28$$

$$\sum_{i=1}^7 k_i^3 = 0$$

$$\sum_{i=1}^7 k_i^4 = 196$$

$$\sum_{i=1}^7 y_i = 1352$$

$$\sum_{i=1}^7 k_i y_i = 162$$

$$\sum_{i=1}^7 k_i^2 y_i = 4286$$

We have

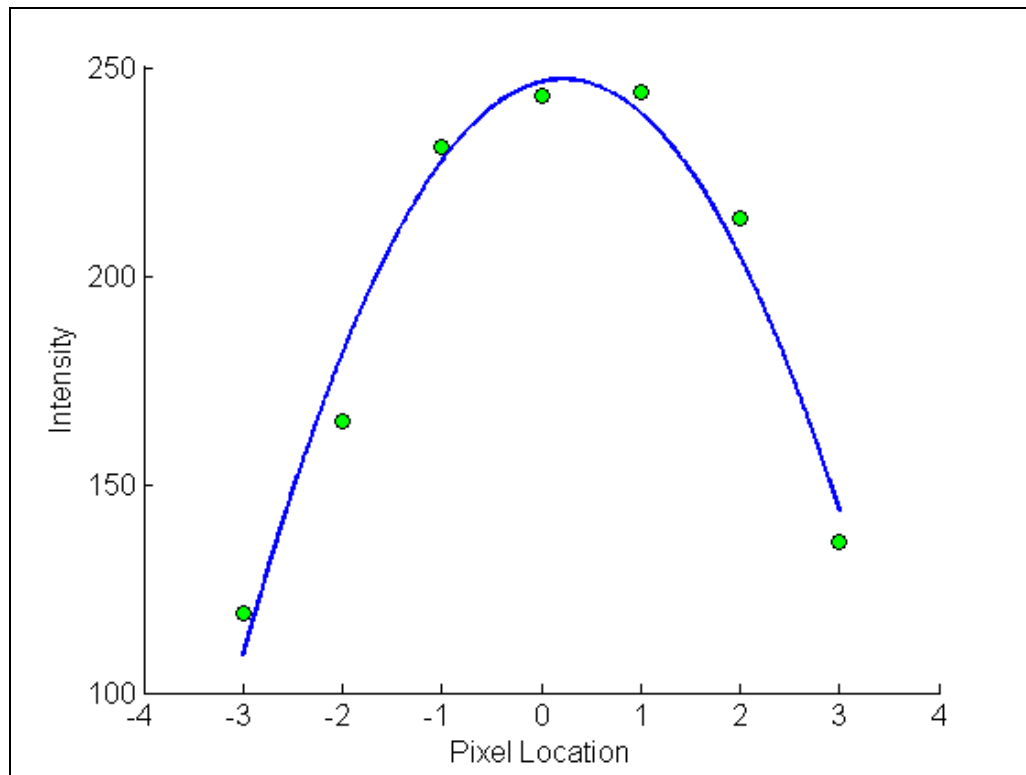
$$\begin{bmatrix} 7 & 0 & 28 \\ 0 & 28 & 0 \\ 28 & 0 & 196 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 1352 \\ 162 \\ 4286 \end{bmatrix}$$

Solve the above system of simultaneous linear equations, we get

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 246.57 \\ 5.7857 \\ -13.357 \end{bmatrix}$$

The polynomial regression model is

$$\begin{aligned} P &= a_0 + a_1 m + a_2 m^2 \\ &= 246.57 + 5.7857m - 13.357m^2 \end{aligned}$$



**Figure 1** Second order polynomial regression model for intensity as a function of pixel location.