Chapter 04.07
LU Decomposition – More Examples
Electrical Engineering

Example 1

Three-phase loads are common in AC systems. When the system is balanced the analysis can be simplified to a single equivalent circuit model. However, when it is unbalanced the only practical solution involves the solution of simultaneous linear equations. In one model the following equations need to be solved.

$$\begin{bmatrix} 0.7460 & -0.4516 & 0.0100 & -0.0080 & 0.0100 & -0.0080 \\ 0.4516 & 0.7460 & 0.0080 & 0.0100 & 0.0080 & 0.0100 \\ 0.0100 & -0.0080 & 0.7787 & -0.5205 & 0.0100 & -0.0080 \\ 0.0080 & 0.0100 & 0.5205 & 0.7787 & 0.0080 & 0.0100 \\ 0.0100 & -0.0080 & 0.0100 & -0.0080 & 0.8080 & -0.6040 \\ 0.0080 & 0.0100 & 0.0080 & 0.0100 & 0.6040 & 0.8080 \end{bmatrix} \begin{bmatrix} I_{ar} \\ I_{ai} \\ I_{br} \\ I_{bi} \\ I_{cr} \\ I_{ci} \end{bmatrix} = \begin{bmatrix} 120 \\ 0.000 \\ -60.00 \\ -103.9 \\ -60.00 \\ 103.9 \end{bmatrix}$$

Find the values of I_{ar}, I_{ai}, I_{br}, I_{bi}, I_{cr}, and I_{ci} using LU decomposition.

Solution

$$[A] = [L][U] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \ell_{21} & 1 & 0 & 0 & 0 & 0 \\ \ell_{31} & \ell_{32} & 1 & 0 & 0 & 0 \\ \ell_{41} & \ell_{42} & \ell_{43} & 1 & 0 & 0 \\ \ell_{51} & \ell_{52} & \ell_{53} & \ell_{54} & 1 & 0 \\ \ell_{61} & \ell_{62} & \ell_{63} & \ell_{64} & \ell_{65} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} & u_{14} & u_{15} & u_{16} \\ 0 & u_{22} & u_{23} & u_{24} & u_{25} & u_{26} \\ 0 & 0 & u_{33} & u_{34} & u_{35} & u_{36} \\ 0 & 0 & 0 & u_{44} & u_{45} & u_{46} \\ 0 & 0 & 0 & 0 & u_{55} & u_{56} \\ 0 & 0 & 0 & 0 & 0 & u_{66} \end{bmatrix}$$

The $[U]$ matrix is the same as the one found at the end of the forward elimination steps of the naïve Gauss elimination method.

Forward Elimination of Unknowns

Since there are six equations, there will be five steps of forward elimination of unknowns.

<u>First step</u>

Divide Row 1 by 0.7460 and multiply it by 0.4516, that is, multiply Row 1 by $0.4516/0.7460 = 0.60536$.

$\text{Row } 1 \times (0.60536) =$

$$[0.4516 \quad -0.27338 \quad 0.0060536 \quad -0.0048429 \quad 0.0060536 \quad -0.0048429] \qquad [72.643]$$

Subtract the result from Row 2 to get

$$\begin{bmatrix} 0.7460 & -0.4516 & 0.0100 & -0.0080 & 0.0100 & -0.0080 \\ 0 & 1.0194 & 0.0019464 & 0.014843 & 0.0019464 & 0.014843 \\ 0.0100 & -0.0080 & 0.7787 & -0.5205 & 0.0100 & -0.0080 \\ 0.0080 & 0.0100 & 0.5205 & 0.7787 & 0.0080 & 0.0100 \\ 0.0100 & -0.0080 & 0.0100 & -0.0080 & 0.8080 & -0.6040 \\ 0.0080 & 0.0100 & 0.0080 & 0.0100 & 0.6040 & 0.8080 \end{bmatrix} \begin{bmatrix} I_{ar} \\ I_{ai} \\ I_{br} \\ I_{bi} \\ I_{cr} \\ I_{ci} \end{bmatrix} = \begin{bmatrix} 120 \\ -72.643 \\ -60.00 \\ -103.9 \\ -60.00 \\ 103.9 \end{bmatrix}$$

Divide Row 1 by 0.7460 and multiply it by 0.0100, that is, multiply Row 1 by $0.0100/0.7460 = 0.013405$.

$\text{Row }1 \times (0.013405) =$

$[0.0100 \quad -0.0060536 \quad 0.00013405 \quad -0.00010724 \quad 0.00013405 \quad -0.00010724] \qquad [1.6086]$

Subtract the result from Row 3 to get

$$\begin{bmatrix} 0.7460 & -0.4516 & 0.0100 & -0.0080 & 0.0100 & -0.0080 \\ 0 & 1.0194 & 0.0019464 & 0.014843 & 0.0019464 & 0.014843 \\ 0 & -0.0019464 & 0.77857 & -0.52061 & 0.0098660 & -0.0078928 \\ 0.0080 & 0.0100 & 0.5205 & 0.7787 & 0.0080 & 0.0100 \\ 0.0100 & -0.0080 & 0.0100 & -0.0080 & 0.8080 & -0.6040 \\ 0.0080 & 0.0100 & 0.0080 & 0.0100 & 0.6040 & 0.8080 \end{bmatrix} \begin{bmatrix} I_{ar} \\ I_{ai} \\ I_{br} \\ I_{bi} \\ I_{cr} \\ I_{ci} \end{bmatrix} = \begin{bmatrix} 120 \\ -72.643 \\ -61.609 \\ -103.9 \\ -60.00 \\ 103.9 \end{bmatrix}$$

Divide Row 1 by 0.7460 and multiply it by 0.0080, that is, multiply Row 1 by $0.0080/0.7460 = 0.010724$.

$\text{Row }1 \times (0.010724) =$

$[0.0080 \quad -0.0048429 \quad 0.00010724 \quad -8.5791\times10^{-5} \quad 0.00010724 \quad -8.5791\times10^{-5}] \qquad [1.2869]$

Subtract the result from Row 4 to get

$$\begin{bmatrix} 0.7460 & -0.4516 & 0.0100 & -0.0080 & 0.0100 & -0.0080 \\ 0 & 1.0194 & 0.0019464 & 0.014843 & 0.0019464 & 0.014843 \\ 0 & -0.0019464 & 0.77857 & -0.52061 & 0.0098660 & -0.0078928 \\ 0 & 0.014843 & 0.52039 & 0.77879 & 0.0078928 & 0.010086 \\ 0.0100 & -0.0080 & 0.0100 & -0.0080 & 0.8080 & -0.6040 \\ 0.0080 & 0.0100 & 0.0080 & 0.0100 & 0.6040 & 0.8080 \end{bmatrix} \begin{bmatrix} I_{ar} \\ I_{ai} \\ I_{br} \\ I_{bi} \\ I_{cr} \\ I_{ci} \end{bmatrix} = \begin{bmatrix} 120 \\ -72.643 \\ -61.609 \\ -105.19 \\ -60.00 \\ 103.9 \end{bmatrix}$$

Divide Row 1 by 0.7460 and multiply it by 0.0100, that is, multiply Row 1 by $0.0100/0.7460 = 0.013405$.

$\text{Row }1 \times (0.013405) =$

$[0.0100 \quad -0.0060536 \quad 0.00013405 \quad -0.00010724 \quad 0.00013405 \quad -0.00010724] \qquad [1.6086]$

Subtract the result from Row 5 to get

$$\begin{bmatrix} 0.7460 & -0.4516 & 0.0100 & -0.0080 & 0.0100 & -0.0080 \\ 0 & 1.0194 & 0.0019464 & 0.014843 & 0.0019464 & 0.014843 \\ 0 & -0.0019464 & 0.77857 & -0.52061 & 0.0098660 & -0.0078928 \\ 0 & 0.014843 & 0.52039 & 0.77879 & 0.0078928 & 0.010086 \\ 0 & -0.0019464 & 0.0098660 & -0.0078928 & 0.80787 & -0.60389 \\ 0.0080 & 0.0100 & 0.0080 & 0.0100 & 0.6040 & 0.8080 \end{bmatrix} \begin{bmatrix} I_{ar} \\ I_{ai} \\ I_{br} \\ I_{bi} \\ I_{cr} \\ I_{ci} \end{bmatrix} = \begin{bmatrix} 120 \\ -72.643 \\ -61.609 \\ -105.19 \\ -61.609 \\ 103.9 \end{bmatrix}$$

Divide Row 1 by 0.7460 and multiply it by 0.0080, that is, multiply Row 1 by $0.0080/0.7460 = 0.010724$.

$\text{Row } 1 \times (0.010724) =$

$$\begin{bmatrix} 0.0080 & -0.0048429 & 0.00010724 & -8.5791\times10^{-5} & 0.00010724 & -8.5791\times10^{-5} \end{bmatrix} \quad \begin{bmatrix} 1.2869 \end{bmatrix}$$

Subtract the result from Row 6 to get

$$\begin{bmatrix} 0.7460 & -0.4516 & 0.0100 & -0.0080 & 0.0100 & -0.0080 \\ 0 & 1.0194 & 0.0019464 & 0.014843 & 0.0019464 & 0.014843 \\ 0 & -0.0019464 & 0.77857 & -0.52039 & 0.0098660 & -0.0078928 \\ 0 & 0.014843 & 0.52039 & 0.77879 & 0.0078928 & 0.010086 \\ 0 & -0.0019464 & 0.0098660 & -0.0078928 & 0.80787 & -0.60389 \\ 0 & 0.014843 & 0.0078928 & 0.010086 & 0.60389 & 0.80809 \end{bmatrix} \begin{bmatrix} I_{ar} \\ I_{ai} \\ I_{br} \\ I_{bi} \\ I_{cr} \\ I_{ci} \end{bmatrix} = \begin{bmatrix} 120 \\ -72.643 \\ -61.609 \\ -105.19 \\ -61.609 \\ 102.61 \end{bmatrix}$$

<u>Second step</u>

Divide Row 2 by 1.0194 and multiply it by -0.0019464, that is, multiply Row 2 by $-0.0019464/1.0194 = -0.0019094$.

$\text{Row } 2 \times (-0.0019094) =$

$$\begin{bmatrix} 0 & -0.0019464 & -3.7164\times10^{-6} & -2.8341\times10^{-5} & -3.7164\times10^{-6} & -2.8341\times10^{-5} \end{bmatrix} \quad \begin{bmatrix} 0.13870 \end{bmatrix}$$

Subtract the result from Row 3 to get

$$\begin{bmatrix} 0.7460 & -0.4516 & 0.0100 & -0.0080 & 0.0100 & -0.0080 \\ 0 & 1.0194 & 0.0019464 & 0.014843 & 0.0019464 & 0.014843 \\ 0 & 0 & 0.77857 & -0.52036 & 0.0098697 & -0.0078644 \\ 0 & 0.014843 & 0.52039 & 0.77879 & 0.0078928 & 0.010086 \\ 0 & -0.0019464 & 0.0098660 & -0.0078928 & 0.80787 & -0.60389 \\ 0 & 0.014843 & 0.0078928 & 0.010086 & 0.60389 & 0.80809 \end{bmatrix} \begin{bmatrix} I_{ar} \\ I_{ai} \\ I_{br} \\ I_{bi} \\ I_{cr} \\ I_{ci} \end{bmatrix} = \begin{bmatrix} 120 \\ -72.643 \\ -61.747 \\ -105.19 \\ -61.609 \\ 102.61 \end{bmatrix}$$

Divide Row 2 by 1.0194 and multiply it by 0.014843, that is, multiply Row 2 by $0.014843/1.0194 = 0.014561$.

$\text{Row } 2 \times (0.014561) =$

$$\begin{bmatrix} 0 & 0.014843 & 2.8341\times10^{-5} & 0.00021612 & 2.8341\times10^{-5} & 0.00021612 \end{bmatrix} \quad \begin{bmatrix} -1.0577 \end{bmatrix}$$

Subtract the result from Row 4 to get

$$\begin{bmatrix} 0.7460 & -0.4516 & 0.0100 & -0.0080 & 0.0100 & -0.0080 \\ 0 & 1.0194 & 0.0019464 & 0.014843 & 0.0019464 & 0.014843 \\ 0 & 0 & 0.77857 & -0.52036 & 0.0098697 & -0.0078644 \\ 0 & 0 & 0.52036 & 0.77857 & 0.0078644 & 0.0098697 \\ 0 & -0.0019464 & 0.0098660 & -0.0078928 & 0.80787 & -0.60389 \\ 0 & 0.014843 & 0.0078928 & 0.010086 & 0.60389 & 0.80809 \end{bmatrix} \begin{bmatrix} I_{ar} \\ I_{ai} \\ I_{br} \\ I_{bi} \\ I_{cr} \\ I_{ci} \end{bmatrix} = \begin{bmatrix} 120 \\ -72.643 \\ -61.747 \\ -104.13 \\ -61.609 \\ 102.61 \end{bmatrix}$$

Divide Row 2 by 1.0194 and multiply it by -0.0019464, that is, multiply Row 2 by $-0.0019464/1.0194 = -0.0019094$.

Row $2 \times (-0.0019094) =$

$$\begin{bmatrix} 0 & -0.0019464 & -3.7164\times10^{-6} & -2.8341\times10^{-5} & -3.7164\times10^{-6} & -2.8341\times10^{-5} \end{bmatrix} \quad \begin{bmatrix} 0.13870 \end{bmatrix}$$

Subtract the result from Row 5 to get

$$\begin{bmatrix} 0.7460 & -0.4516 & 0.0100 & -0.0080 & 0.0100 & -0.0080 \\ 0 & 1.0194 & 0.0019464 & 0.014843 & 0.0019464 & 0.014843 \\ 0 & 0 & 0.77857 & -0.52036 & 0.0098697 & -0.0078644 \\ 0 & 0 & 0.52036 & 0.77857 & 0.0078644 & 0.0098697 \\ 0 & 0 & 0.0098697 & -0.0078644 & 0.80787 & -0.60386 \\ 0 & 0.014843 & 0.0078928 & 0.010086 & 0.60389 & 0.80809 \end{bmatrix} \begin{bmatrix} I_{ar} \\ I_{ai} \\ I_{br} \\ I_{bi} \\ I_{cr} \\ I_{ci} \end{bmatrix} = \begin{bmatrix} 120 \\ -72.643 \\ -61.747 \\ -104.13 \\ -61.747 \\ 102.61 \end{bmatrix}$$

Divide Row 2 by 1.0194 and multiply it by 0.014843, that is, multiply Row 2 by $0.014843/1.0194 = 0.014561$.

Row $2 \times (0.014561) =$

$$\begin{bmatrix} 0 & 0.014843 & 2.8341\times10^{-5} & 0.00021612 & 2.8341\times10^{-5} & 0.00021612 \end{bmatrix} \quad \begin{bmatrix} -1.0577 \end{bmatrix}$$

Subtract the result from Row 6 to get

$$\begin{bmatrix} 0.7460 & -0.4516 & 0.0100 & -0.0080 & 0.0100 & -0.0080 \\ 0 & 1.0194 & 0.0019464 & 0.014843 & 0.0019464 & 0.014843 \\ 0 & 0 & 0.77857 & -0.52036 & 0.0098697 & -0.0078644 \\ 0 & 0 & 0.52036 & 0.77857 & 0.0078644 & 0.0098697 \\ 0 & 0 & 0.0098697 & -0.0078644 & 0.80787 & -0.60386 \\ 0 & 0 & 0.0078644 & 0.0098697 & 0.60386 & 0.80787 \end{bmatrix} \begin{bmatrix} I_{ar} \\ I_{ai} \\ I_{br} \\ I_{bi} \\ I_{cr} \\ I_{ci} \end{bmatrix} = \begin{bmatrix} 120 \\ -72.643 \\ -61.747 \\ -104.13 \\ -61.747 \\ 103.67 \end{bmatrix}$$

<u>Third step</u>

Divide Row 3 by 0.77857 and multiply it by 0.52036, that is, multiply Row 3 by $0.52036/0.77857 = 0.66836$.

Row $3 \times (0.66836) =$

$$\begin{bmatrix} 0 & 0 & 0.52036 & -0.34779 & 0.0065965 & -0.0052563 \end{bmatrix} \quad \begin{bmatrix} -41.269 \end{bmatrix}$$

Subtract the result from Row 4 to get

$$\begin{bmatrix} 0.7460 & -0.4516 & 0.0100 & -0.0080 & 0.0100 & -0.0080 \\ 0 & 1.0194 & 0.0019464 & 0.014843 & 0.0019464 & 0.014843 \\ 0 & 0 & 0.77857 & -0.52036 & 0.0098697 & -0.0078644 \\ 0 & 0 & 0 & 1.1264 & 0.0012679 & 0.015126 \\ 0 & 0 & 0.0098697 & -0.0078644 & 0.80787 & -0.60386 \\ 0 & 0 & 0.0078644 & 0.0098697 & 0.60386 & 0.80787 \end{bmatrix} \begin{bmatrix} I_{ar} \\ I_{ai} \\ I_{br} \\ I_{bi} \\ I_{cr} \\ I_{ci} \end{bmatrix} = \begin{bmatrix} 120 \\ -72.643 \\ -61.747 \\ -62.860 \\ -61.747 \\ 103.67 \end{bmatrix}$$

Divide Row 3 by 0.77857 and multiply it by 0.0098697, that is, multiply Row 3 by $0.0098697/0.77857 = 0.012677$.

$\text{Row } 3 \times (0.012677) =$

$$\begin{bmatrix} 0 & 0 & 0.0098697 & -0.0065965 & 0.00012511 & -9.9695\times10^{-5} \end{bmatrix} \quad \begin{bmatrix} -0.78275 \end{bmatrix}$$

Subtract the result from Row 5 to get

$$\begin{bmatrix} 0.7460 & -0.4516 & 0.0100 & -0.0080 & 0.0100 & -0.0080 \\ 0 & 1.0194 & 0.0019464 & 0.014843 & 0.0019464 & 0.014843 \\ 0 & 0 & 0.77857 & -0.52036 & 0.0098697 & -0.0078644 \\ 0 & 0 & 0 & 1.1264 & 0.0012679 & 0.015126 \\ 0 & 0 & 0 & -0.0012679 & 0.80774 & -0.60376 \\ 0 & 0 & 0.0078644 & 0.0098697 & 0.60386 & 0.80787 \end{bmatrix} \begin{bmatrix} I_{ar} \\ I_{ai} \\ I_{br} \\ I_{bi} \\ I_{cr} \\ I_{ci} \end{bmatrix} = \begin{bmatrix} 120 \\ -72.643 \\ -61.747 \\ -62.860 \\ -60.965 \\ 103.67 \end{bmatrix}$$

Divide Row 3 by 0.77857 and multiply it by 0.0078644, that is, multiply Row 3 by $0.0078644/0.77857 = 0.010101$.

$\text{Row } 3 \times (0.010101) =$

$$\begin{bmatrix} 0 & 0 & 0.0078644 & -0.0052563 & 9.9695\times10^{-5} & -7.9439\times10^{-5} \end{bmatrix} \quad \begin{bmatrix} -0.62372 \end{bmatrix}$$

Subtract the result from Row 6 to get

$$\begin{bmatrix} 0.7460 & -0.4516 & 0.0100 & -0.0080 & 0.0100 & -0.0080 \\ 0 & 1.0194 & 0.0019464 & 0.014843 & 0.0019464 & 0.014843 \\ 0 & 0 & 0.77857 & -0.52036 & 0.0098697 & -0.0078644 \\ 0 & 0 & 0 & 1.1264 & 0.0012679 & 0.015126 \\ 0 & 0 & 0 & -0.0012679 & 0.80774 & -0.60376 \\ 0 & 0 & 0 & 0.015126 & 0.60376 & 0.80795 \end{bmatrix} \begin{bmatrix} I_{ar} \\ I_{ai} \\ I_{br} \\ I_{bi} \\ I_{cr} \\ I_{ci} \end{bmatrix} = \begin{bmatrix} 120 \\ -72.643 \\ -61.747 \\ -62.860 \\ -60.965 \\ 104.29 \end{bmatrix}$$

<u>Fourth step</u>

Divide Row 4 by 1.1264 and multiply it by -0.0012679, that is, multiply Row 4 by $-0.0012679/1.1264 = -0.0011257$.

$\text{Row } 4 \times (-0.0011257) =$

$$\begin{bmatrix} 0 & 0 & 0 & -0.0012679 & -1.4273\times10^{-6} & -1.7027\times10^{-5} \end{bmatrix} \quad \begin{bmatrix} 0.070761 \end{bmatrix}$$

Subtract the result from Row 5 to get

$$\begin{bmatrix} 0.7460 & -0.4516 & 0.0100 & -0.0080 & 0.0100 & -0.0080 \\ 0 & 1.0194 & 0.0019464 & 0.014843 & 0.0019464 & 0.014843 \\ 0 & 0 & 0.77857 & -0.52036 & 0.0098697 & -0.0078644 \\ 0 & 0 & 0 & 1.1264 & 0.0012679 & 0.015126 \\ 0 & 0 & 0 & 0 & 0.80775 & -0.60375 \\ 0 & 0 & 0 & 0.015126 & 0.60376 & 0.80795 \end{bmatrix} \begin{bmatrix} I_{ar} \\ I_{ai} \\ I_{br} \\ I_{bi} \\ I_{cr} \\ I_{ci} \end{bmatrix} = \begin{bmatrix} 120 \\ -72.643 \\ -61.747 \\ -62.860 \\ -61.035 \\ 104.29 \end{bmatrix}$$

Divide Row 4 by 1.1264 and multiply it by 0.015126, that is, multiply Row 4 by $0.015126/1.1264 = 0.013429$.

$\text{Row } 4 \times (0.013429) =$

$$\begin{bmatrix} 0 & 0 & 0 & 0.015126 & 1.7027\times10^{-5} & 0.00016196 \end{bmatrix} \quad \begin{bmatrix} -0.67308 \end{bmatrix}$$

Subtract the result from Row 6 to get

$$\begin{bmatrix} 0.7460 & -0.4516 & 0.0100 & -0.0080 & 0.0100 & -0.0080 \\ 0 & 1.0194 & 0.0019464 & 0.014843 & 0.0019464 & 0.014843 \\ 0 & 0 & 0.77857 & -0.52036 & 0.0098697 & -0.0078644 \\ 0 & 0 & 0 & 1.1264 & 0.0012679 & 0.015126 \\ 0 & 0 & 0 & 0 & 0.80775 & -0.60375 \\ 0 & 0 & 0 & 0 & 0.60375 & 0.80775 \end{bmatrix} \begin{bmatrix} I_{ar} \\ I_{ai} \\ I_{br} \\ I_{bi} \\ I_{cr} \\ I_{ci} \end{bmatrix} = \begin{bmatrix} 120 \\ -72.643 \\ -61.747 \\ -62.860 \\ -61.035 \\ 104.97 \end{bmatrix}$$

<u>Fifth step</u>

Divide Row 5 by 0.80775 and multiply it by 0.60375, that is, multiply Row 5 by $0.60375/0.80775 = 0.74745$.

$\text{Row } 5 \times (0.74741) =$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0.60375 & -0.45127 \end{bmatrix} \quad \begin{bmatrix} -45.621 \end{bmatrix}$$

Subtract the result from Row 6 to get

$$\begin{bmatrix} 0.7460 & -0.4516 & 0.0100 & -0.0080 & 0.0100 & -0.0080 \\ 0 & 1.0194 & 0.0019464 & 0.014843 & 0.0019464 & 0.014843 \\ 0 & 0 & 0.77857 & -0.52036 & 0.0098697 & -0.0078644 \\ 0 & 0 & 0 & 1.1264 & 0.0012679 & 0.015126 \\ 0 & 0 & 0 & 0 & 0.80775 & -0.60375 \\ 0 & 0 & 0 & 0 & 0 & 1.2590 \end{bmatrix} \begin{bmatrix} I_{ar} \\ I_{ai} \\ I_{br} \\ I_{bi} \\ I_{cr} \\ I_{ci} \end{bmatrix} = \begin{bmatrix} 120 \\ -72.643 \\ -61.747 \\ -62.860 \\ -61.035 \\ 150.76 \end{bmatrix}$$

The coefficient matrix after the completion of the forward elimination steps is the $[U]$ matrix.

$$[U]=\begin{bmatrix} 0.7460 & -0.4516 & 0.0100 & -0.0080 & 0.0100 & -0.0080 \\ 0 & 1.0194 & 0.0019464 & 0.014843 & 0.0019464 & 0.014843 \\ 0 & 0 & 0.77857 & -0.52036 & 0.0098697 & -0.0078644 \\ 0 & 0 & 0 & 1.1264 & 0.0012679 & 0.015126 \\ 0 & 0 & 0 & 0 & 0.80775 & -0.60375 \\ 0 & 0 & 0 & 0 & 0 & 1.2590 \end{bmatrix}$$

Now find $[L]$.

$$[L]=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \ell_{21} & 1 & 0 & 0 & 0 & 0 \\ \ell_{31} & \ell_{32} & 1 & 0 & 0 & 0 \\ \ell_{41} & \ell_{42} & \ell_{43} & 1 & 0 & 0 \\ \ell_{51} & \ell_{52} & \ell_{53} & \ell_{54} & 1 & 0 \\ \ell_{61} & \ell_{62} & \ell_{63} & \ell_{64} & \ell_{65} & 1 \end{bmatrix}$$

From Step 1 of the forward elimination process

$$\ell_{21}=\frac{0.4516}{0.7460}=0.60536$$

$$\ell_{31}=\frac{0.01}{0.7460}=0.013405$$

$$\ell_{41}=\frac{0.008}{0.7460}=0.010724$$

$$\ell_{51}=\frac{0.01}{0.7460}=0.013405$$

$$\ell_{61}=\frac{0.008}{0.7460}=0.010724$$

From Step 2 of the forward elimination process

$$\ell_{32}=\frac{-0.0019464}{1.0194}=-0.0019094$$

$$\ell_{42}=\frac{0.014843}{1.0194}=0.014561$$

$$\ell_{52}=\frac{-0.0019464}{1.0194}=-0.0019094$$

$$\ell_{62}=\frac{0.014843}{1.0194}=0.014561$$

From Step 3 of the forward elimination process

$$\ell_{43}=\frac{0.52036}{0.77857}=0.66836$$

$$\ell_{53}=\frac{0.0098697}{0.77857}=0.012677$$

$$\ell_{63} = \frac{0.0078644}{0.77857} = 0.01010$$

From Step 4 of the forward elimination process

$$\ell_{54} = \frac{-0.0012679}{1.1264} = -0.0011257$$

$$\ell_{64} = \frac{0.015126}{1.1264} = 0.013429$$

From Step 5 of the forward elimination process

$$\ell_{65} = \frac{0.60375}{0.80775} = 0.74745$$

Hence

$$[L] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0.60536 & 1 & 0 & 0 & 0 & 0 \\ 0.013405 & -0.0019094 & 1 & 0 & 0 & 0 \\ 0.010724 & 0.014561 & 0.66836 & 1 & 0 & 0 \\ 0.013405 & -0.0019094 & 0.012677 & -0.0011257 & 1 & 0 \\ 0.010724 & 0.014561 & 0.01010 & 0.013429 & 0.74745 & 1 \end{bmatrix}$$

Now that $[L]$ and $[U]$ are known, solve $[L][Z]=[C]$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0.60536 & 1 & 0 & 0 & 0 & 0 \\ 0.013405 & -0.0019094 & 1 & 0 & 0 & 0 \\ 0.010724 & 0.014561 & 0.66836 & 1 & 0 & 0 \\ 0.013405 & -0.0019094 & 0.012677 & -0.0011257 & 1 & 0 \\ 0.010724 & 0.014561 & 0.01010 & 0.013429 & 0.74745 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \end{bmatrix} = \begin{bmatrix} 120 \\ 0.000 \\ -60.00 \\ -103.9 \\ -60.00 \\ 103.9 \end{bmatrix}$$

This provides the six equations

$$z_1 = 120$$

$$0.60536z_1 + z_2 = 0.000$$

$$0.013405z_1 + (-0.0019094)z_2 + z_3 = -60.00$$

$$0.010724z_1 + 0.014561z_2 + 0.66836z_3 + z_4 = -103.9$$

$$0.013405z_1 + (-0.0019094)z_2 + 0.012677z_3 + (-0.0011257)z_4 + z_5 = -60.00$$

$$0.010724z_1 + 0.014561z_2 + 0.01010z_3 + 0.013429z_4 + 0.74745z_5 + z_6 = 103.9$$

Forward substitution starting from the first equation gives

$$z_1 = 120$$

Substituting the value of z_1 into the second equation,

$$\begin{aligned} z_2 &= -0.60536z_1 \\ &= -0.60536(120) \\ &= -72.643 \end{aligned}$$

Substituting the values of z_1 and z_2 into the third equation,

$$\begin{aligned} z_3 &= -60.00 - 0.013405 z_1 - (-0.0019094) z_2 \\ &= -60.00 - 0.013405(120) - (-0.0019094)(-72.643) \\ &= -61.747 \end{aligned}$$

Substituting the values of z_1, z_2, and z_3 into the fourth equation,

$$\begin{aligned} z_4 &= -103.9 - 0.010724 z_1 - 0.014561 z_2 - 0.66836 z_3 \\ &= -103.9 - 0.010724(120) - 0.014561(-72.643) - 0.66836(-61.747) \\ &= -62.860 \end{aligned}$$

Substituting the values of z_1, z_2, z_3, and z_4 into the fifth equation,

$$\begin{aligned} z_5 &= -60.00 - 0.013405 z_1 - (-0.0019094) z_2 - 0.012677 z_3 - (-0.0011257) z_4 \\ &= -60.00 - 0.013405(120) - (-0.0019094)(-72.643) \\ &\quad - 0.012677(-61.747) - (-0.0011257)(-62.860) \\ &= -61.035 \end{aligned}$$

Substituting the values of z_1, z_2, z_3, z_4, and z_5 into the sixth equation,

$$\begin{aligned} z_6 &= 103.9 - 0.010724 z_1 - 0.014561 z_2 - 0.01010 z_3 - 0.013429 z_4 - 0.74745 z_5 \\ &= 103.9 - 0.010724(120) - 0.014561(-72.643) - 0.01010(-61.747) \\ &\quad - 0.013429(-62.860) - 0.74745(-61.035) \\ &= 150.76 \end{aligned}$$

Hence

$$[Z] = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \end{bmatrix} = \begin{bmatrix} 120 \\ -72.643 \\ -61.747 \\ -62.860 \\ -61.035 \\ 150.76 \end{bmatrix}$$

Now solve $[U][I] = [Z]$.

$$\begin{bmatrix} 0.7460 & -0.4516 & 0.0100 & -0.0080 & 0.0100 & -0.0080 \\ 0 & 1.0194 & 0.0019464 & 0.014843 & 0.0019464 & 0.014843 \\ 0 & 0 & 0.77857 & -0.52036 & 0.0098697 & -0.0078644 \\ 0 & 0 & 0 & 1.1264 & 0.0012679 & 0.015126 \\ 0 & 0 & 0 & 0 & 0.80775 & -0.60375 \\ 0 & 0 & 0 & 0 & 0 & 1.2590 \end{bmatrix} \begin{bmatrix} I_{ar} \\ I_{ai} \\ I_{br} \\ I_{bi} \\ I_{cr} \\ I_{ci} \end{bmatrix} = \begin{bmatrix} 120 \\ -72.643 \\ -61.747 \\ -62.860 \\ -61.035 \\ 150.76 \end{bmatrix}$$

This provides the six equations for $[I]$.

$$0.7460 I_{ar} + (-0.4516) I_{ai} + 0.0100 I_{br} + (-0.0080) I_{bi} + 0.0100 I_{cr} + (-0.0080) I_{ci} = 120$$

$$1.0194 I_{ai} + 0.0019464 I_{br} + 0.014843 I_{bi} + 0.0019464 I_{cr} + 0.014843 I_{ci} = -72.643$$

$$0.77857 I_{br} + (-0.52036) I_{bi} + 0.0098697 I_{cr} + (-0.0078644) I_{ci} = -61.747$$
$$1.1264 I_{bi} + 0.0012679 I_{cr} + 0.015126 I_{ci} = -62.860$$
$$0.80775 I_{cr} + (-0.60375) I_{ci} = -61.035$$
$$1.2590 I_{ci} = 150.76$$

From the sixth equation

$$1.2590 I_{ci} = 150.76$$
$$I_{ci} = \frac{150.76}{1.2590}$$
$$= 119.74$$

Substituting the value of I_{ci} into the fifth equation,

$$0.80775 I_{cr} + (-0.60375) I_{ci} = -61.035$$
$$I_{cr} = \frac{-61.035 - (-0.60375) I_{ci}}{0.80775}$$
$$= 13.940$$

Substituting the values of I_{cr} and I_{ci} into the fourth equation,

$$1.1264 I_{bi} + 0.0012679 I_{cr} + 0.015126 I_{ci} = -62.860$$
$$I_{bi} = \frac{-62.860 - 0.0012679 I_{cr} - 0.015126 I_{ci}}{1.1264}$$
$$= -57.432$$

Substituting the values of I_{bi}, I_{cr}, and I_{ci} into the third equation,

$$0.77857 I_{br} + (-0.52036) I_{bi} + 0.0098697 I_{cr} + (-0.0078644) I_{ci} = -61.747$$
$$I_{br} = \frac{-61.747 - (-0.52036) I_{bi} - 0.0098697 I_{cr} - (-0.0078644) I_{ci}}{0.77857}$$
$$= -116.66$$

Substituting the values of I_{br}, I_{bi}, I_{cr}, and I_{ci} into the second equation,

$$1.0194 I_{ai} + 0.0019464 I_{br} + 0.014843 I_{bi} + 0.0019464 I_{cr} + 0.014843 I_{ci} = -72.643$$
$$I_{ai} = \frac{-72.643 - 0.0019464 I_{br} - 0.014843 I_{bi} - 0.0019464 I_{cr} - 0.014843 I_{ci}}{1.0194}$$
$$= -71.973$$

Substituting the values of I_{ai}, I_{br}, I_{bi}, I_{cr}, and I_{ci} into the first equation,

$$0.7460 I_{ar} + (-0.4516) I_{ai} + 0.0100 I_{br} + (-0.0080) I_{bi} + 0.0100 I_{cr} + (-0.0080) I_{ci} = 120$$
$$I_{ar} = \frac{120 - (-0.4516) I_{ai} - 0.0100 I_{br} - (-0.0080) I_{bi} - 0.0100 I_{cr} - (-0.0080) I_{ci}}{0.7460}$$
$$= 119.33$$

The solution vector is

$$\begin{bmatrix} I_{ar} \\ I_{ai} \\ I_{br} \\ I_{bi} \\ I_{cr} \\ I_{ci} \end{bmatrix} = \begin{bmatrix} 119.33 \\ -71.973 \\ -116.66 \\ -57.432 \\ 13.940 \\ 119.74 \end{bmatrix}$$

SIMULTANEOUS LINEAR EQUATIONS	
Topic	LU Decomposition – More Examples
Summary	Examples of LU decomposition
Major	Electrical Engineering
Authors	Autar Kaw
Date	August 8, 2009
Web Site	http://numericalmethods.eng.usf.edu