Chapter 02.03
Differentiation of Discrete Functions-More Examples
Electrical Engineering

Example 1

To increase the reliability and life of a switch, one needs to turn the switch off as close to the zero-crossing as possible. To find this time of zero-crossing, the value of $E(t)/E'(t)$ is to be found at all times given in Table 1, where $E(t)$ is the voltage and t is the time. To keep the problem simple, you are asked to find the approximate value of $E(t)/E'(t)$ at $t = 10$.

Table 1 Voltage as a function of time.

Time, t (s)	Voltage, $E(t)$ (V)		Time, t (s)	Voltage, $E(t)$ (V)
1	0.62161		13	–0.210796
2	0.362358		14	0.087499
3	0.070737		15	0.377978
4	–0.227202		16	0.634693
5	–0.504846		17	0.834713
6	–0.737394		18	0.96017
7	–0.904072		19	0.999859
8	–0.989992		20	0.950233
9	–0.98748		21	0.815725
10	–0.896758		22	0.608351
11	–0.725932		23	0.346635
12	–0.490261		24	0.053955

Use the forward divided difference approximation of the first derivative to calculate $E(t)/E'(t)$ at $t = 10$. Use a step size of $\Delta t = 1$.

Solution

$$E'(t_i) \approx \frac{E(t_{i+1}) - E(t_i)}{\Delta t}$$

$$t_i = 10$$

$$t_{i+1} = 11$$

$$\begin{aligned}\Delta t &= t_{i+1} - t_i \\ &= 11 - 10 \\ &= 1\end{aligned}$$

$$\begin{aligned}E'(10) &\approx \frac{E(11) - E(10)}{1} \\ &= \frac{-0.725932 - (-0.896758)}{1} \\ &= 0.170826 \text{ V/s}\end{aligned}$$

$$\begin{aligned}\frac{E(10)}{E'(10)} &= \frac{-0.896758}{0.170826} \\ &= -5.2495 \text{ s}\end{aligned}$$

Example 2

To increase the reliability and life of a switch, one needs to turn the switch off as close to the zero-crossing as possible. To find this time of zero-crossing, the value of $E(t)/E'(t)$ is to be found at all times given in Table 2, where $E(t)$ is the voltage and t is the time. To keep the problem simple, you are asked to find the approximate value of $E(t)/E'(t)$ at $t = 10$.

Table 2 Voltage as a function of time.

Time, t (s)	Voltage, $E(t)$ (V)		Time, t (s)	Voltage, $E(t)$ (V)
1	0.62161		13	–0.210796
2	0.362358		14	0.087499
3	0.070737		15	0.377978
4	–0.227202		16	0.634693
5	–0.504846		17	0.834713
6	–0.737394		18	0.96017
7	–0.904072		19	0.999859
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9	–0.98748		21	0.815725
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12	–0.490261		24	0.053955

Using a third order polynomial interpolant for Voltage, find the value of $E(t)/E'(t)$ at $t = 10$.

Solution

For a third order polynomial interpolation (also called cubic interpolation), we choose the voltage given by

$$E(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

Since we want to find the voltage at $t = 10$, and we are using a third order polynomial, we need to choose the four points closest to $t = 10$ that also bracket $t = 10$ to evaluate it.

The four points are $t_0 = 8$, $t_1 = 9$, $t_2 = 10$ and $t_3 = 11$.

$$t_0 = 8, \quad E(t_0) = -0.989992$$
$$t_1 = 9, \quad E(t_1) = -0.98748$$
$$t_2 = 10, \quad E(t_2) = -0.896758$$
$$t_3 = 11, \quad E(t_3) = -0.725932$$

such that

$$E(8) = -0.989992 = a_0 + a_1(8) + a_2(8)^2 + a_3(8)^3$$
$$E(9) = -0.98748 = a_0 + a_1(9) + a_2(9)^2 + a_3(9)^3$$
$$E(10) = -0.896758 = a_0 + a_1(10) + a_2(10)^2 + a_3(10)^3$$
$$E(11) = -0.725932 = a_0 + a_1(11) + a_2(11)^2 + a_3(11)^3$$

Writing the four equations in matrix form, we have

$$\begin{bmatrix} 1 & 8 & 64 & 512 \\ 1 & 9 & 81 & 729 \\ 1 & 10 & 100 & 1000 \\ 1 & 11 & 121 & 1331 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -0.989992 \\ -0.98748 \\ -0.896758 \\ -0.725932 \end{bmatrix}$$

Solving the above gives

$$a_0 = 3.1382$$
$$a_1 = -1.0742$$
$$a_2 = 0.080582$$
$$a_3 = -0.001351$$

Hence

$$E(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$
$$= 3.1382 - 1.0742t + 0.080582t^2 - 0.0013510t^3, \quad 8 \le t \le 11$$

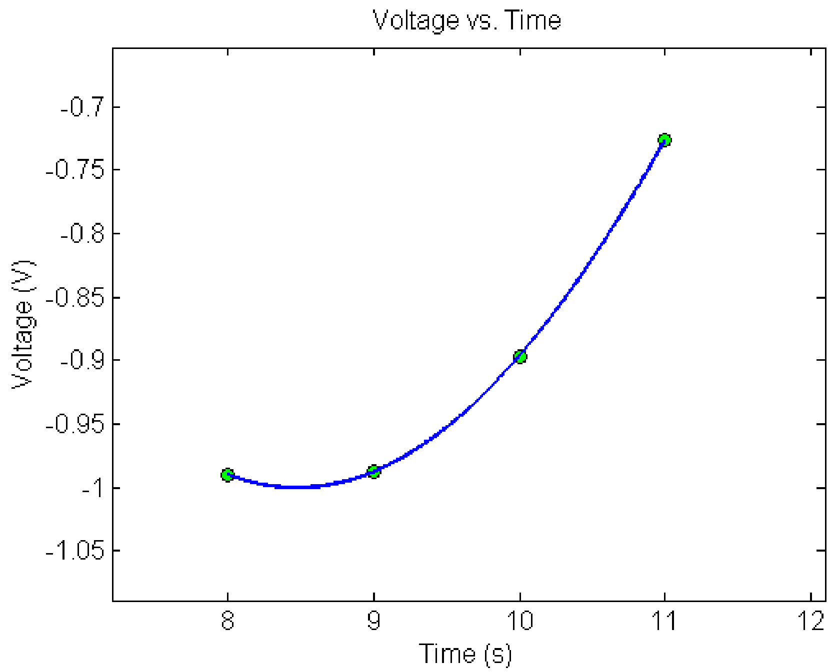

Figure 1 Graph of voltage of the switch vs. time.

The derivative of voltage at $t = 10$ is given by

$$E'(10) = \frac{d}{dt}E(t)\Big|_{t=10}$$

Given that $E(t) = 3.1382 - 1.0742t + 0.080582t^2 - 0.0013510t^3$, $8 \le t \le 11$,

$$\begin{aligned}E'(t) &= \frac{d}{dt}E(t)\\ &= \frac{d}{dt}\left(3.13812 - 1.0742t + 0.080582t^2 - 0.0013510t^3\right)\\ &= -1.0742 + 0.16116t - 0.004053t^2,\ 8 \le t \le 11\end{aligned}$$

$$\begin{aligned}E'(10) &= -1.0742 + 0.16116(10) - 0.004053(10)^2\\ &= 0.13210 \text{ V}/s\end{aligned}$$

$$\begin{aligned}\frac{E(10)}{E'(10)} &= \frac{-0.896758}{0.13210}\\ &= -6.7872 \text{ s}\end{aligned}$$

Example 3

To increase the reliability and life of a switch, one needs to turn the switch off as close to the zero-crossing as possible. To find this time of zero-crossing, the value of $E(t)/E'(t)$ is to be found at all times given in Table 3, where $E(t)$ is the voltage and t is the time. To keep the problem simple, you are asked to find the approximate value of $E(t)/E'(t)$ at $t = 10$.

Table 3 Voltage as a function of time.

Time, t (s)	Voltage, $E(t)$ (V)		Time, t (s)	Voltage, $E(t)$ (V)
1	0.62161		13	–0.210796
2	0.362358		14	0.087499
3	0.070737		15	0.377978
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12	–0.490261		24	0.053955

Use second order Lagrangian polynomial interpolation to calculate $E(t)/E'(t)$ at $t=10$.

Solution

For second order Lagrangian polynomial interpolation, we choose the voltage given by

$$E(t)=\left(\frac{t-t_1}{t_0-t_1}\right)\left(\frac{t-t_2}{t_0-t_2}\right)E(t_0)+\left(\frac{t-t_0}{t_1-t_0}\right)\left(\frac{t-t_2}{t_1-t_2}\right)E(t_1)+\left(\frac{t-t_0}{t_2-t_0}\right)\left(\frac{t-t_1}{t_2-t_1}\right)E(t_2)$$

Since we want to find the voltage at $t=10$, and we are using a second order Lagrangian polynomial, we need to choose the three points closest to $t=10$ that also bracket $t=10$ to evaluate it. The three points are $t_0=9$, $t_1=10$, and $t_2=11$.

Differentiating the above equation gives

$$E'(t)=\frac{2t-(t_1+t_2)}{(t_0-t_1)(t_0-t_2)}E(t_0)+\frac{2t-(t_0+t_2)}{(t_1-t_0)(t_1-t_2)}E(t_1)+\frac{2t-(t_0+t_1)}{(t_2-t_0)(t_2-t_1)}E(t_2)$$

Hence

$$\begin{aligned}E'(10)&=\frac{2(10)-(10+11)}{(9-10)(9-11)}(-0.98748)+\frac{2(10)-(9+11)}{(10-9)(10-11)}(-0.896758)\\&\quad+\frac{2(10)-(9+10)}{(11-9)(11-10)}(-0.725932)\\&=-0.5(-0.98748)+0(-0.896758)+0.5(-0.725932)\\&=0.13077\ \text{V/s}\end{aligned}$$

$$\begin{aligned}\frac{E(10)}{E'(10)}&=\frac{-0.896758}{0.13077}\\&=-6.8573\ \text{s}\end{aligned}$$

DIFFERENTIATION	
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