Chapter 08.02
Euler’s Method for Ordinary Differential Equations-
More Examples
Computer Engineering

Example 1
A rectifier-based power supply requires a capacitor to temporarily store power when the
rectified waveform from the AC source drops below the target voltage. To properly size this
capacitor a first-order ordinary differential equation must be solved. For a particular power
supply, with a capacitor of $150 \mu F$, the ordinary differential equation to be solved is

$$\frac{dv(t)}{dt} = \frac{1}{150 \times 10^{-6}} \left\{-0.1 + \max\left\{\frac{18 \cos(120\pi t)}{0.04} - 2 - v(t), 0\right\}\right\}$$

$v(0) = 0$

Using Euler’s method, find the voltage across the capacitor at $t = 0.00004$ s. Use step size
$h = 0.00002$ s.

Solution

$$\frac{dv}{dt} = \frac{1}{150 \times 10^{-6}} \left\{-0.1 + \max\left\{\frac{18 \cos(120\pi t)}{0.04} - 2 - v, 0\right\}\right\}$$

$$f(t,v) = \frac{1}{150 \times 10^{-6}} \left\{-0.1 + \max\left\{\frac{18 \cos(120\pi t)}{0.04} - 2 - v, 0\right\}\right\}$$

The Euler’s method reduces to

$v_{i+1} = v_i + f(t_i, v_i)h$

For $i = 0$, $t_0 = 0$, $v_0 = 0$

$v_1 = v_0 + f(t_0, v_0)h$

$= 0 + f(0,0)0.00002$

$= \frac{1}{150 \times 10^{-6}} \left\{-0.1 + \max\left\{\frac{18 \cos(120\pi (0))}{0.04} - 2 - (0), 0\right\}\right\}0.00002$

$= 0 + (2.666 \times 10^8)0.00002$

$= 53.320$ V

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\( v_1 \) is the approximate voltage at
\[
    t = t_1 = t_0 + h = 0 + 0.00002 = 0.00002 \text{ s}
\]
\[
    v(0.00002) \approx v_1 = 53.320 \text{ V}
\]
For \( i = 1, t_i = 0.00002, v_1 = 53.320 \)
\[
    v_2 = v_1 + f(t_i, v_i)h
    = 53.320 + f(0.00002, 53.320) 0.00002
    = 53.320 + \frac{1}{150 \times 10^{-6}} \left( -0.1 + \max \left\{ \frac{18 \cos(120\pi(0.00002))}{-2 - (53.320)/0.04}, 0 \right\} \right) 0.00002
    = 53.320 + (-0.000015000) 0.00002
    = 53.307 \text{ V}
\]
\( v_2 \) is the approximate voltage at
\[
    t = t_2 = t_1 + h = 0.00002 + 0.00002 = 0.00004 \text{ s}
\]
\[
    v(0.00004) \approx v_2 = 53.307 \text{ V}
\]
Figure 1 compares the exact solution of \( v(0.00004) = 15.974 \text{ V} \) with the numerical solution from Euler’s method for the step size of \( h = 0.00004 \text{ s} \).
The problem was solved again using smaller step sizes. The results are given below in Table 1.

Table 1  Voltage at 0.00004 seconds as a function of step size, $h$.

| Step size, $h$ | $v(0.00004)$ | $E_t$ | $|\varepsilon_i|\%$ |
|---------------|--------------|-------|-----------------|
| 0.00004       | 106.64       | -90.667 | 567.59         |
| 0.00002       | 53.307       | -37.333 | 233.71         |
| 0.00001       | 26.640       | -10.666 | 66.771        |
| 0.000005      | 15.996       | -0.02199 | 0.13766   |
| 0.0000025     | 15.993       | -0.019125 | 0.11972   |

Figure 2 shows how the voltage varies as a function of time for different step sizes.

While the values of the calculated voltage at $t = 0.00004s$ as a function of step size are plotted in Figure 3.
Figure 3 Effect of step size in Euler’s method.