

Chapter 07.02

Trapezoidal Rule for Integration-More Examples

Computer Engineering

Example 1

Human vision has the remarkable ability to infer 3D shapes from 2D images. The intriguing question is: can we replicate some of these abilities on a computer? Yes, it can be done and to do this, integration of vector fields is required. The following integral needs to be integrated.

$$I = \int_0^{100} f(x) dx$$

Where,

$$\begin{aligned} f(x) &= 0, \quad 0 < x < 30 \\ &= -9.1688 \times 10^{-6} x^3 + 2.7961 \times 10^{-3} x^2 - 2.8487 \times 10^{-1} x + 9.6778, \quad 30 \leq x \leq 172 \\ &= 0, \quad 172 < x < 200 \end{aligned}$$

- Use single segment Trapezoidal rule to find the value of the integral.
- Find the true error, E_t , for part (a).
- Find the absolute relative true error for part (a).

Solution

$$a) \quad I \approx (b-a) \left[\frac{f(a) + f(b)}{2} \right], \text{ where}$$

$$\begin{aligned} a &= 0 \\ b &= 100 \end{aligned}$$

$$\begin{aligned} f(x) &= 0, \quad 0 < x < 30 \\ &= -9.1688 \times 10^{-6} x^3 + 2.7961 \times 10^{-3} x^2 - 2.8487 \times 10^{-1} x + 9.6778, \quad 30 \leq x \leq 172 \\ &= 0, \quad 172 < x < 200 \end{aligned}$$

$$f(0) = 0$$

$$\begin{aligned} f(100) &= -9.1688 \times 10^{-6} \times (100)^3 + 2.7961 \times 10^{-3} \times (100)^2 - 2.8487 \times 10^{-1} \times (100) + 9.6778 \\ &= -0.017000 \end{aligned}$$

$$I \approx (100 - 0) \left[\frac{0 + (-0.017)}{2} \right]$$

$$\approx -0.85000$$

b) The exact value of the above integral is found using Maple for calculating the true error and relative true error.

$$I = \int_0^{100} f(x)dx$$

$$= 60.793$$

so the true error is

$$E_t = \text{True Value} - \text{Approximate Value}$$

$$= 60.793 - (-0.85000)$$

$$= 61.643$$

c) The absolute relative true error, $|\epsilon_t|$, would then be

$$|\epsilon_t| = \left| \frac{\text{True Error}}{\text{True Value}} \right| \times 100\%$$

$$= \left| \frac{60.793 - (-0.850900)}{60.793} \right| \times 100\%$$

$$= 101.40\%$$

Example 2

Human vision has the remarkable ability to infer 3D shapes from 2D images. The intriguing question is: can we replicate some of these abilities on a computer? Yes, it can be done and to do this, integration of vector fields is required. The following integral needs to be integrated.

$$I = \int_0^{100} f(x)dx$$

where

$$f(x) = 0, \quad 0 < x < 30$$

$$= -9.1688 \times 10^{-6} x^3 + 2.7961 \times 10^{-3} x^2 - 2.8487 \times 10^{-1} x + 9.6778, \quad 30 \leq x \leq 172$$

$$= 0, \quad 172 < x < 200$$

- Use 2-segment Trapezoidal rule to find the value of the integral
- Find the true error, E_t , for part (a).
- Find the absolute relative true error, $|\epsilon_t|$, for part (a).

Solution

$$a) \quad I = \frac{b-a}{2n} \left[f(a) + 2 \left\{ \sum_{i=1}^{n-1} f(a+ih) \right\} + f(b) \right]$$

$$n = 2$$

$$a = 0$$

$$b = 100$$

$$\begin{aligned}
 h &= \frac{b-a}{n} \\
 &= \frac{100-0}{2} \\
 &= 50
 \end{aligned}$$

$$\begin{aligned}
 f(x) &= 0, \quad 0 < x < 30 \\
 &= -9.1688 \times 10^{-6} x^3 + 2.7961 \times 10^{-3} x^2 - 2.8487 \times 10^{-1} x + 9.6778, \quad 30 \leq x \leq 172 \\
 &= 0, \quad 172 < x < 200
 \end{aligned}$$

$$\begin{aligned}
 I &\approx \frac{100-0}{2(2)} \left[f(0) + 2 \left\{ \sum_{i=1}^{2-1} f(a+ih) \right\} + f(100) \right] \\
 &\approx \frac{100}{4} \left[f(0) + 2 \sum_{i=1}^1 f(0+1 \times 50) + f(100) \right] \\
 &\approx \frac{100}{4} [f(0) + 2f(50) + f(100)] \\
 &\approx \frac{100}{4} [0 + 2(1.2784) + (-0.017000)] \\
 &\approx 63.497
 \end{aligned}$$

Since

$$\begin{aligned}
 f(0) &= 0 \\
 f(100) &= -9.1688 \times 10^{-6} \times (100)^3 + 2.7961 \times 10^{-3} \times (100)^2 - 2.8487 \times 10^{-1} \times (100) + 9.6778 \\
 &= -0.017000 \\
 f(50) &= -9.1688 \times 10^{-6} \times (50)^3 + 2.7961 \times 10^{-3} \times (50)^2 - 2.8487 \times 10^{-1} \times (50) + 9.6778 \\
 &= 1.2784
 \end{aligned}$$

b) The exact value of the above integral is found using Maple for calculating the true error and relative true error.

$$\begin{aligned}
 I &= \int_0^{100} f(x) dx \\
 &= 60.793
 \end{aligned}$$

so the true error is

$$\begin{aligned}
 E_t &= \text{True Value} - \text{Approximate Value} \\
 &= 60.793 - 63.497 \\
 &= -2.7049
 \end{aligned}$$

c) The absolute relative true error, $|\epsilon_t|$, would then be

$$|\epsilon_t| = \left| \frac{\text{True Error}}{\text{True Value}} \right| \times 100 \%$$

$$= \left| \frac{60.793 - 63.497}{60.793} \right| \times 100\%$$

$$= 4.4494\%$$

Table 1 Values obtained using multiple-segment Trapezoidal rule for
 $f(x) = 0, 0 < x < 30$
 $= -9.1688 \times 10^{-6} x^3 + 2.7961 \times 10^{-3} x^2 - 2.8487 \times 10^{-1} x + 9.6778, 30 \leq x \leq 172$
 $= 0, 172 < x < 200$

n	Value	E_t	$ \epsilon_t \%$	$ \epsilon_a \%$
1	-0.85000	61.643	101.40	---
2	63.498	-2.7049	4.4494	101.34
3	111.26	-50.465	83.011	42.927
4	36.062	24.731	40.681	208.52
5	58.427	2.3652	3.8906	38.279
6	77.769	-16.977	27.925	24.870
7	42.528	18.265	30.044	82.866
8	55.754	5.0388	8.2885	23.722