Chapter 07.02  
Trapezoidal Rule for Integration-More Examples  
Computer Engineering

Example 1

Human vision has the remarkable ability to infer 3D shapes from 2D images. The intriguing question is: can we replicate some of these abilities on a computer? Yes, it can be done and to do this, integration of vector fields is required. The following integral needs to be integrated.

\[ I = \int_{0}^{100} f(x) \, dx \]

Where,

\[ f(x) = 0, \quad 0 < x < 30 \]
\[ = -9.1688 \times 10^{-6} x^3 + 2.7961 \times 10^{-3} x^2 - 2.8487 \times 10^{-1} x + 9.6778, \quad 30 \leq x \leq 172 \]
\[ = 0, \quad 172 < x < 200 \]

a) Use single segment Trapezoidal rule to find the value of the integral.
b) Find the true error, \( E_t \), for part (a).
c) Find the absolute relative true error for part (a).

Solution

a) \[ I \approx (b - a) \left[ \frac{f(a) + f(b)}{2} \right], \text{ where} \]
\[ a = 0 \]
\[ b = 100 \]
\[ f(x) = 0, \quad 0 < x < 30 \]
\[ = -9.1688 \times 10^{-6} x^3 + 2.7961 \times 10^{-3} x^2 - 2.8487 \times 10^{-1} x + 9.6778, \quad 30 \leq x \leq 172 \]
\[ = 0, \quad 172 < x < 200 \]
\[ f(0) = 0 \]
\[ f(100) = -9.1688 \times 10^{-6} \times (100)^3 + 2.7961 \times 10^{-3} \times (100)^2 - 2.8487 \times 10^{-1} \times (100) + 9.6778 \]
\[ = -0.017000 \]
\[ I \approx (100 - 0) \left[ \frac{0 + (-0.017)}{2} \right] \]
\[ \approx -0.85000 \]
b) The exact value of the above integral is found using Maple for calculating the true error and relative true error.

\[ I = \int_{0}^{100} f(x)dx \]

\[ = 60.793 \]

so the true error is

\[ E_t = True\ Value - Approximate\ Value \]

\[ = 60.793 - (-0.8500) \]

\[ = 61.643 \]

c) The absolute relative true error, |\( \varepsilon \)|, would then be

\[ |\varepsilon| = \left| \frac{True\ Error}{True\ Value} \right| \times 100\% \]

\[ = \left| \frac{60.793 - (-0.850900)}{60.793} \right| \times 100\% \]

\[ = 101.40\% \]

**Example 2**

Human vision has the remarkable ability to infer 3D shapes from 2D images. The intriguing question is: can we replicate some of these abilities on a computer? Yes, it can be done and to do this, integration of vector fields is required. The following integral needs to integrated.

\[ I = \int_{0}^{100} f(x)dx \]

where

\[ f(x) = 0, \ 0 < x < 30 \]

\[ = -9.1688 \times 10^{-6} x^3 + 2.7961 \times 10^{-3} x^2 - 2.8487 \times 10^{-1} x + 9.6778, \ 30 \leq x \leq 172 \]

\[ = 0, \ 172 < x < 200 \]

a) Use 2-segment Trapezoidal rule to find the value of the integral

b) Find the true error, \( E_t \), for part (a).

c) Find the absolute relative true error, |\( \varepsilon \)|, for part (a).

**Solution**

a) \[ I = \frac{b-a}{2n} \left[ f(a) + 2 \sum_{i=1}^{n-1} f(a + ih) \right] + f(b) \]

\[ n = 2 \]

\[ a = 0 \]

\[ b = 100 \]
\[ h = \frac{b-a}{n} = \frac{100 - 0}{2} = 50 \]

\[ f(x) = 0, \ 0 < x < 30 \]
\[ = -9.1688 \times 10^{-6} x^3 + 2.7961 \times 10^{-3} x^2 - 2.8487 \times 10^{-1} x + 9.6778, \ 30 \leq x \leq 172 \]
\[ = 0, \ 172 < x < 200 \]

\[ I \approx \frac{100 - 0}{2(2)} \left[ f(0) + 2 \left\{ \sum_{i=1}^{2-1} f(a + ih) \right\} + f(100) \right] \]
\[ \approx \frac{100}{4} \left[ f(0) + 2 \sum_{i=1}^{1} f(0 + 1 \times 50) + f(100) \right] \]
\[ \approx \frac{100}{4} \left[ f(0) + 2 f(50) + f(100) \right] \]
\[ \approx \frac{100}{4} \left[ 0 + 2(1.2784) + (-0.017000) \right] \]
\[ \approx 63.497 \]

Since
\[ f(0) = 0 \]
\[ f(100) = -9.1688 \times 10^{-6} \times (100)^3 + 2.7961 \times 10^{-3} \times (100)^2 - 2.8487 \times 10^{-1} \times (100) + 9.6778 \]
\[ = -0.017000 \]
\[ f(50) = -9.1688 \times 10^{-6} \times (50)^3 + 2.7961 \times 10^{-3} \times (50)^2 - 2.8487 \times 10^{-1} \times (50) + 9.6778 \]
\[ = 1.2784 \]

b) The exact value of the above integral is found using Maple for calculating the true error and relative true error.
\[ I = \int_{0}^{100} f(x)dx \]
\[ = 60.793 \]

so the true error is
\[ E_t = True \ Value - Approximate \ Value \]
\[ = 60.793 - 63.497 \]
\[ = -2.7049 \]

c) The absolute relative true error, \(|\varepsilon_i|\), would then be
\[ |\varepsilon_i| = \left| \frac{True \ Error}{True \ Value} \right| \times 100 \% \]
\[
\frac{60.793 - 63.497}{60.793} \times 100\% = 4.4494\%
\]

**Table 1** Values obtained using multiple-segment Trapezoidal rule for $f(x) = 0$, $0 < x < 30$

\[f(x) = -9.1688 \times 10^{-6} x^3 + 2.7961 \times 10^{-3} x^2 - 2.8487 \times 10^{-1} x + 9.6778, \quad 30 \leq x \leq 172\]

\[= 0, \quad 172 < x < 200\]

| $n$ | Value  | $E_i$  | $|\varepsilon_i|\%$ | $|\varepsilon_e|\%$ |
|-----|--------|--------|----------------------|----------------------|
| 1   | -0.85000 | 61.643 | 101.40               | ---                  |
| 2   | 63.498   | -2.7049| 4.4494               | 101.34               |
| 3   | 111.26   | -50.465| 83.011               | 42.927               |
| 4   | 36.062   | 24.731 | 40.681               | 208.52               |
| 5   | 58.427   | 2.3652 | 3.8906               | 38.279               |
| 6   | 77.769   | -16.977| 27.925               | 24.870               |
| 7   | 42.528   | 18.265 | 30.044               | 82.866               |
| 8   | 55.754   | 5.0388 | 8.2885               | 23.722               |