Chapter 07.02
Trapezoidal Rule for Integration-More Examples
Computer Engineering

Example 1

Human vision has the remarkable ability to infer 3D shapes from 2D images. The intriguing question is: can we replicate some of these abilities on a computer? Yes, it can be done and to do this, integration of vector fields is required. The following integral needs to integrated.

$$I = \int_0^{100} f(x)dx$$

Where,

$$\begin{aligned} f(x) &= 0,\ 0 < x < 30 \\ &= -9.1688\times 10^{-6} x^3 \ + \ 2.7961\times 10^{-3} x^2 \ - \ 2.8487\times 10^{-1} x \ + \ 9.6778, \ \ 30 \le x \le 172 \\ &= 0, \ \ 172 < x < 200 \end{aligned}$$

a) Use single segment Trapezoidal rule to find the value of the integral.
b) Find the true error, E_t, for part (a).
c) Find the absolute relative true error for part (a).

Solution

a) $$I \approx (b-a)\left[\frac{f(a)+f(b)}{2}\right], \text{ where}$$

$$a = 0$$
$$b = 100$$

$$\begin{aligned} f(x) &= 0,\ 0 < x < 30 \\ &= -9.1688\times 10^{-6} x^3 \ + \ 2.7961\times 10^{-3} x^2 \ - \ 2.8487\times 10^{-1} x \ + \ 9.6778, \ \ 30 \le x \le 172 \\ &= 0, \ \ 172 < x < 200 \end{aligned}$$

$$f(0) = 0$$

$$\begin{aligned} f(100) &= -9.1688\times 10^{-6}\times(100)^3 + 2.7961\times 10^{-3}\times(100)^2 - 2.8487\times 10^{-1}\times(100) + 9.6778 \\ &= -0.017000 \end{aligned}$$

$$\begin{aligned} I &\approx (100-0)\left[\frac{0+(-0.017)}{2}\right] \\ &\approx -0.85000 \end{aligned}$$

b) The exact value of the above integral is found using Maple for calculating the true error and relative true error.

$$I = \int_0^{100} f(x)dx$$
$$= 60.793$$

so the true error is

$$E_t = True\ Value - Approximate\ Value$$
$$= 60.793 - (-0.85000)$$
$$= 61.643$$

c) The absolute relative true error, $|\in_t|$, would then be

$$|\in_t| = \left|\frac{\text{True Error}}{\text{True Value}}\right| \times 100\,\%$$

$$= \left|\frac{60.793 - (-0.850900)}{60.793}\right| \times 100\,\%$$
$$= 101.40\,\%$$

Example 2

Human vision has the remarkable ability to infer 3D shapes from 2D images. The intriguing question is: can we replicate some of these abilities on a computer? Yes, it can be done and to do this, integration of vector fields is required. The following integral needs to integrated.

$$I = \int_0^{100} f(x)dx$$

where

$$f(x) = 0,\ 0 < x < 30$$
$$= -9.1688 \times 10^{-6} x^3 + 2.7961 \times 10^{-3} x^2 - 2.8487 \times 10^{-1} x + 9.6778,\ 30 \le x \le 172$$
$$= 0,\ 172 < x < 200$$

a) Use 2-segment Trapezoidal rule to find the value of the integral
b) Find the true error, E_t, for part (a).
c) Find the absolute relative true error, $|\in_t|$,for part (a).

Solution

a) $$I = \frac{b-a}{2n}\left[f(a) + 2\left\{\sum_{i=1}^{n-1} f(a+ih)\right\} + f(b)\right]$$

$$n = 2$$
$$a = 0$$
$$b = 100$$

$$h = \frac{b-a}{n}$$
$$= \frac{100-0}{2}$$
$$= 50$$

$$f(x) = 0,\ 0 < x < 30$$
$$= -9.1688\times10^{-6}x^3 + 2.7961\times10^{-3}x^2 - 2.8487\times10^{-1}x + 9.6778,\ \ 30 \le x \le 172$$
$$= 0,\ \ 172 < x < 200$$
$$I \approx \frac{100-0}{2(2)}\left[f(0) + 2\left\{\sum_{i=1}^{2-1} f(a+ih)\right\} + f(100)\right]$$
$$\approx \frac{100}{4}\left[f(0) + 2\sum_{i=1}^{1} f(0+1\times50) + f(100)\right]$$
$$\approx \frac{100}{4}[f(0) + 2f(50) + f(100)]$$
$$\approx \frac{100}{4}[0 + 2(1.2784) + (-0.017000)]$$
$$\approx 63.497$$

Since

$$f(0) = 0$$
$$f(100) = -9.1688\times10^{-6}\times(100)^3 + 2.7961\times10^{-3}\times(100)^2 - 2.8487\times10^{-1}\times(100) + 9.6778$$
$$= -0.017000$$
$$f(50) = -9.1688\times10^{-6}\times(50)^3 + 2.7961\times10^{-3}\times(50)^2 - 2.8487\times10^{-1}\times(50) + 9.6778$$
$$= 1.2784$$

b) The exact value of the above integral is found using Maple for calculating the true error and relative true error.

$$I = \int_0^{100} f(x)dx$$
$$= 60.793$$

so the true error is

$$E_t = True\ Value - Approximate\ Value$$
$$= 60.793 - 63.497$$
$$= -2.7049$$

c) The absolute relative true error, $|\in_t|$, would then be

$$|\in_t| = \left|\frac{\text{True Error}}{\text{True Value}}\right| \times 100\,\%$$

$$= \left| \frac{60.793 - 63.497}{60.793} \right| \times 100\,\%$$
$$= 4.4494\,\%$$

Table 1 Values obtained using multiple-segment Trapezoidal rule for
$f(x) = 0,\ 0 < x < 30$

$= -9.1688 \times 10^{-6} x^3 \ + \ 2.7961 \times 10^{-3} x^2 \ - \ 2.8487 \times 10^{-1} x \ + \ 9.6778, \ \ 30 \le x \le 172$

$= 0, \ \ 172 < x < 200$

n	Value	E_t	$\lvert \in_t \rvert\,\%$	$\lvert \in_a \rvert\,\%$
1	−0.85000	61.643	101.40	---
2	63.498	−2.7049	4.4494	101.34
3	111.26	−50.465	83.011	42.927
4	36.062	24.731	40.681	208.52
5	58.427	2.3652	3.8906	38.279
6	77.769	−16.977	27.925	24.870
7	42.528	18.265	30.044	82.866
8	55.754	5.0388	8.2885	23.722