Chapter 04.06
Gaussian Elimination – More Examples
Computer Engineering

Example 1

To infer the surface shape of an object from images taken of a surface from three different directions, one needs to solve the following set of equations.

$$\begin{bmatrix} 0.2425 & 0 & -0.9701 \\ 0 & 0.2425 & -0.9701 \\ -0.2357 & -0.2357 & -0.9428 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 247 \\ 248 \\ 239 \end{bmatrix}$$

The right hand side values are the light intensities from the middle of the images, while the coefficient matrix is dependent on the light source directions with respect to the camera. The unknowns are the incident intensities that will determine the shape of the object.

Find the values of x_1, x_2, and x_3 using naïve Gauss elimination.

Solution

Forward Elimination of Unknowns

Since there are three equations, there will be two steps of forward elimination of unknowns.

First step

Divide Row 1 by 0.2425 and then multiply it by 0, that is, multiply Row 1 by $0/0.2425 = 0$.

$$\text{Row } 1 \times (0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 \end{bmatrix}$$

Subtract the result from Row 2 to get

$$\begin{bmatrix} 0.2425 & 0 & -0.9701 \\ 0 & 0.2425 & -0.9701 \\ -0.2357 & -0.2357 & -0.9428 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 247 \\ 248 \\ 239 \end{bmatrix}$$

Divide Row 1 by 0.2425 and then multiply it by -0.2357, that is, multiply Row 1 by $-0.2357/0.2425 = -0.97196$.

$$\text{Row } 1 \times (-0.97196) = \begin{bmatrix} -0.2357 & 0 & 0.094290 \end{bmatrix} \qquad \begin{bmatrix} -240.07 \end{bmatrix}$$

Subtract the result from Row 3 to get

$$\begin{bmatrix} 0.2425 & 0 & -0.9701 \\ 0 & 0.2425 & -0.9701 \\ 0 & -0.2357 & -1.8857 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 247 \\ 248 \\ 479.07 \end{bmatrix}$$

Second step

We now divide Row 2 by 0.2425 and then multiply by -0.2357, that is, multiply Row 2 by $-0.2357/0.2425 = -0.97196$.

$$\text{Row } 2 \times (-0.97196) = \begin{bmatrix} 0 & -0.2357 & 0.94290 \end{bmatrix} \quad \begin{bmatrix} -241.05 \end{bmatrix}$$

Subtract the result from Row 3 to get

$$\begin{bmatrix} 0.2425 & 0 & -0.9701 \\ 0 & 0.2425 & -0.9701 \\ 0 & 0 & -2.8286 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 247 \\ 248 \\ 720.12 \end{bmatrix}$$

Back substitution

From the third equation,

$$-2.8286 x_3 = 720.12$$

$$x_3 = \frac{720.12}{-2.8286}$$

$$= -254.59$$

Substituting the value of x_3 in the second equation,

$$0.2425 x_2 + (-0.9701) x_3 = 248$$

$$x_2 = \frac{248 - (-0.9701) x_3}{0.2425}$$

$$= \frac{248 - (-0.9701) \times (-254.59)}{0.2425}$$

$$= 4.2328$$

Substituting the values of x_2 and x_3 in the first equation,

$$0.2425 x_1 + 0 x_2 + (-0.9701) x_3 = 247$$

$$x_1 = \frac{247 - 0 x_2 - (-0.9701) x_3}{0.2425}$$

$$= \frac{247 - (-0.9701) \times (-254.59)}{0.2425}$$

$$= 0.10905$$

Hence the solution vector is

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.10905 \\ 4.2328 \\ -254.59 \end{bmatrix}$$

SIMULTANEOUS LINEAR EQUATIONS	
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