Chapter 03.05
Secant Method of Solving a Nonlinear Equation – More Examples
Computer Science

Example 1

To find the inverse of a number a, one can use the equation

$$f(c) = a - \frac{1}{c} = 0$$

where c is the inverse of a.

Use the secant method of finding roots of equations to find the inverse of $a = 2.5$. Conduct three iterations to estimate the root of the above equation. Find the absolute relative approximate error at the end of each iteration and the number of significant digits at least correct at the end of each iteration.

Solution

$$f(c) = a - \frac{1}{c} = 0$$

$$c_{i+1} = c_i - \frac{\left(a - \frac{1}{c_i}\right)(c_i - c_{i-1})}{\left(a - \frac{1}{c_i}\right) - \left(a - \frac{1}{c_{i-1}}\right)}$$

$$= c_i - \frac{\left(a - \frac{1}{c_i}\right)(c_i - c_{i-1})}{\frac{1}{c_{i-1}} - \frac{1}{c_i}}$$

$$= c_i - \frac{\left(a - \frac{1}{c_i}\right)(c_i - c_{i-1})}{\frac{(c_i - c_{i-1})}{c_i c_{i-1}}}$$

$$= c_i - c_i c_{i-1}\left(a - \frac{1}{c_i}\right)$$

$$= c_i - c_{i-1}(ac_i - 1)$$

Let us take the initial guesses of the root of $f(c) = 0$ as $c_{-1} = 0.1$ and $c_0 = 0.6$.

Iteration 1

The estimate of the root is

$$c_1 = c_0 - c_{-1}(ac_0 - 1)$$
$$= 0.6 - (0.1)(2.5(0.6) - 1)$$
$$= 0.55$$

The absolute relative approximate error $|\in_a|$ at the end of Iteration 1 is

$$|\in_a| = \left|\frac{c_1 - c_0}{c_1}\right| \times 100$$
$$= \left|\frac{0.55 - 0.6}{0.55}\right| \times 100$$
$$= 9.0909\%$$

The number of significant digits at least correct is 0, as you need an absolute relative approximate error of less than 5% for one significant digit to be correct in your result.

Iteration 2

The estimate of the root is

$$c_2 = c_1 - c_0(ac_1 - 1)$$
$$= 0.55 - (0.6)(2.5(0.55) - 1)$$
$$= 0.325$$

The absolute relative approximate error $|\in_a|$ at the end of Iteration 2 is

$$|\in_a| = \left|\frac{c_2 - c_1}{c_2}\right| \times 100$$
$$= \left|\frac{0.325 - 0.55}{0.325}\right| \times 100$$
$$= 69.231\%$$

The number of significant digits at least correct is 0.

Iteration 3

The estimate of the root is

$$c_3 = c_2 - c_1(ac_2 - 1)$$
$$= 0.325 - (0.55)(2.5(0.325) - 1)$$
$$= 0.42813$$

The absolute relative approximate error $|\in_a|$ at the end of Iteration 3 is

$$|\in_a| = \left|\frac{c_3 - c_2}{c_3}\right| \times 100$$

$$= \left| \frac{0.42813 - 0.325}{0.42813} \right| \times 100$$

$$= 24.088\%$$

The number of significant digits at least correct is 0.

NONLINEAR EQUATIONS	
Topic	Secant Method-More Examples
Summary	Examples of Secant Method
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