Chapter 06.03
Linear Regression-More Examples
Civil Engineering

Example 1

The coefficient of thermal expansion, α ,of steel is given at discrete values of temperature in Table 1.

Table 1 Coefficient of thermal expansion versus temperature for steel.

Temperature, T	Coefficient of thermal expansion, α
°F	in/in°F
80	6.470×10^{-6}
60	6.360×10^{-6}
40	6.240×10^{-6}
20	6.120×10^{-6}
0	6.000×10^{-6}
−20	5.860×10^{-6}
−40	5.720×10^{-6}
−60	5.580×10^{-6}
−80	5.430×10^{-6}
−100	5.280×10^{-6}
−120	5.090×10^{-6}
−140	4.910×10^{-6}
−160	4.720×10^{-6}
−180	4.520×10^{-6}
−200	4.300×10^{-6}
−220	4.080×10^{-6}
−240	3.830×10^{-6}
−260	3.580×10^{-6}
−280	3.330×10^{-6}
−300	3.070×10^{-6}
−320	2.760×10^{-6}
−340	2.450×10^{-6}

The data is regressed to a first order polynomial.

$$\alpha = k_1 + k_2 T$$

Find the constants k_1 and k_2 of the regression model.

Solution

Table 2 shows the summations needed for the calculation of the constants of the regression model.

Table 2 Tabulation of data for calculation of needed summations.

I	T	α	$T\alpha$	T^2
–	°F	in/in/°F	in/in/°F	$(°F)^2$
1	80	6.470×10^{-6}	5.1760×10^{-4}	6400
2	60	6.360×10^{-6}	3.8160×10^{-4}	3600
3	40	6.240×10^{-6}	2.4960×10^{-4}	1600
4	20	6.120×10^{-6}	1.2240×10^{-4}	400
5	0	6.000×10^{-6}	0.000	0
6	-20	5.860×10^{-6}	-1.1720×10^{-4}	400
7	-40	5.720×10^{-6}	-2.2880×10^{-4}	1600
8	-60	5.580×10^{-6}	-3.3480×10^{-4}	3600
9	-80	5.430×10^{-6}	-4.3440×10^{-4}	6400
10	-100	5.280×10^{-6}	-5.2800×10^{-4}	10000
11	-120	5.090×10^{-6}	-6.1080×10^{-4}	14400
12	-140	4.910×10^{-6}	-6.8740×10^{-4}	19600
13	-160	4.720×10^{-6}	-7.5520×10^{-4}	25600
14	-180	4.520×10^{-6}	-8.1360×10^{-4}	32400
15	-200	4.300×10^{-6}	-8.6000×10^{-4}	40000
16	-220	4.080×10^{-6}	-8.9760×10^{-4}	48400
17	-240	3.830×10^{-6}	-9.1920×10^{-4}	57600
18	-260	3.580×10^{-6}	-9.3080×10^{-4}	67600
19	-280	3.330×10^{-6}	-9.3240×10^{-4}	78400
20	-300	3.070×10^{-6}	-9.2100×10^{-4}	90000
21	-320	2.760×10^{-6}	-8.8320×10^{-4}	102400
22	-340	2.450×10^{-6}	-8.3300×10^{-4}	115600
$\sum_{i=1}^{22}$	-2860	1.0570×10^{-4}	-1.0416×10^{-2}	726000

$$n = 22$$

$$k_2 = \frac{n\sum_{i=1}^{22} T_i\alpha_i - \sum_{i=1}^{22} T_i \sum_{i=1}^{22}\alpha_i}{n\sum_{i=1}^{22} T_i^2 - \left(\sum_{i=1}^{22} T_i\right)^2}$$

$$= \frac{22\left(-1.0416\times10^{-2}\right) - (-2860)\left(1.0570\times10^{-4}\right)}{22(726000) - (-2860)^2}$$

$$= 9.3868\times10^{-9} \text{ in/in/(°F)}^2$$

$$\bar{\alpha} = \frac{\sum_{i=1}^{22}\alpha_i}{n}$$

$$= \frac{1.0570\times10^{-4}}{22}$$

$$= 4.8045\times10^{-6} \text{ in/in°F}$$

$$\bar{T} = \frac{\sum_{i=1}^{22} T_i}{n}$$

$$= \frac{-2860}{22}$$

$$= -130 \text{ °F}$$

$$k_1 = \bar{\alpha} - k_2\bar{T}$$

$$= 4.8045\times10^{-6} - \left(9.3868\times10^{-9}\right)(-130)$$

$$= 6.0248\times10^{-6} \text{ in/in°F}$$

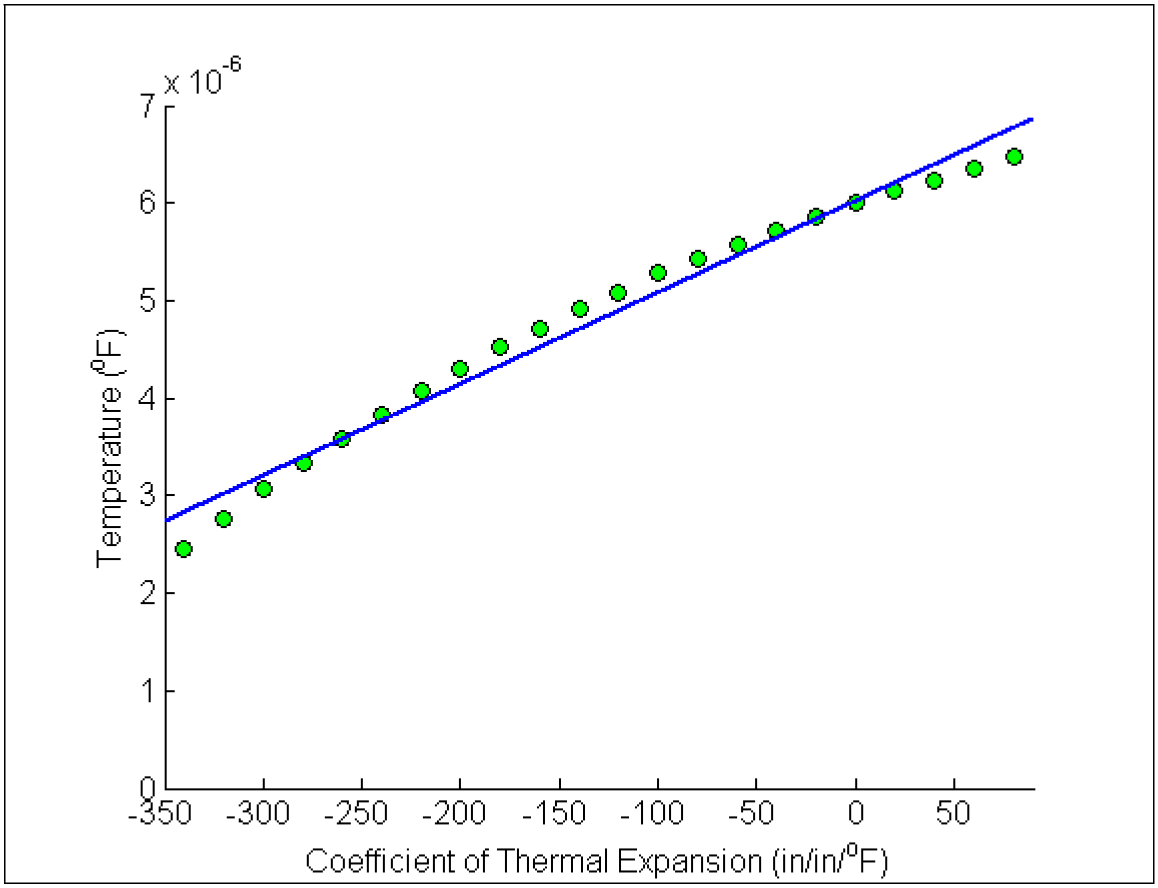

Figure 4 Linear regression of coefficient of thermal expansion vs. temperature data.