Chapter 05.02
Direct Method of Interpolation – More Examples
Chemical Engineering

Example 1

To find how much heat is required to bring a kettle of water to its boiling point, you are asked to calculate the specific heat of water at $61°\text{C}$. The specific heat of water is given as a function of time in Table 1.

Table 1 Specific heat of water as a function of temperature.

Temperature, T $(°\text{C})$	Specific heat, C_p $\left(\frac{\text{J}}{\text{kg}-°\text{C}}\right)$
22	4181
42	4179
52	4186
82	4199
100	4217

Determine the value of the specific heat at $T = 61°\text{C}$ using the direct method of interpolation and a first order polynomial.

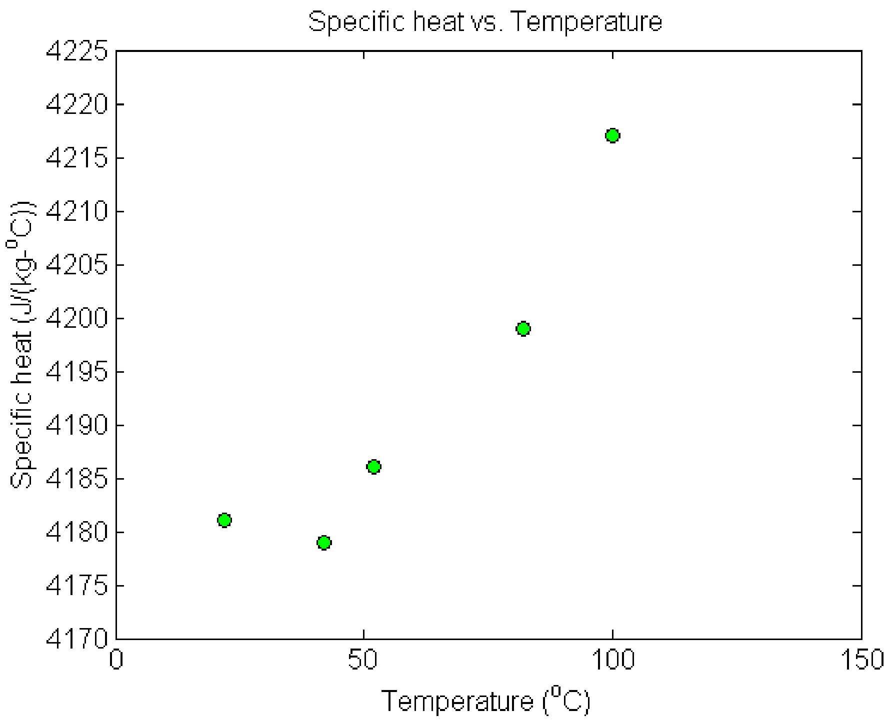

Figure 1 Specific heat of water vs. temperature.

Solution

For first order polynomial interpolation (also called linear interpolation), we choose the specific heat given by

$$C_p(T) = a_0 + a_1 T$$

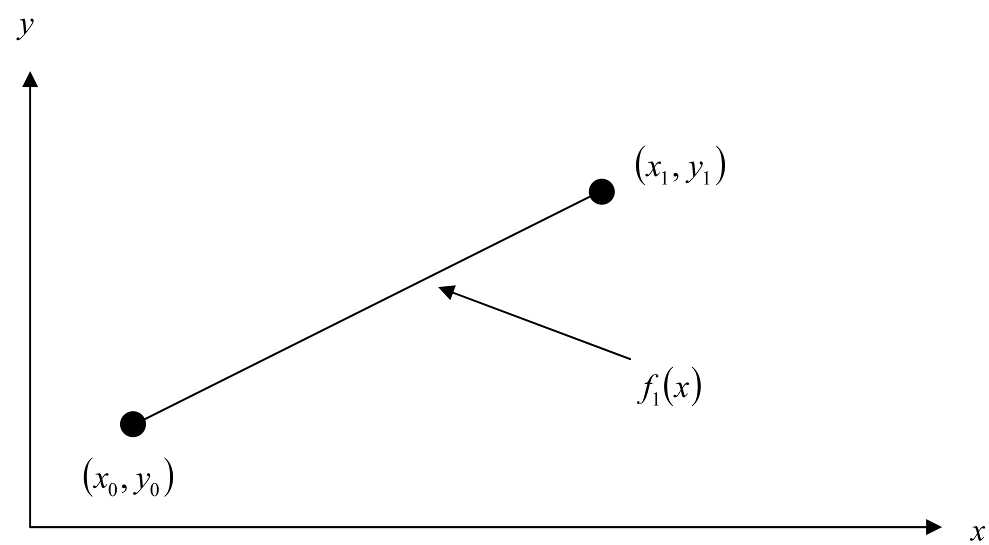

Figure 2 Linear interpolation.

Since we want to find the specific heat at $T = 61°\text{C}$, and we are using a first order polynomial, we need to choose the two data points that are closest to $T = 61°\text{C}$ that also bracket $T = 61°\text{C}$ to evaluate it. The two points are $T_0 = 52$ and $T_1 = 82$.

Then

$$T_0 = 52,\ C_p(T_0) = 4186$$

$$T_1 = 82,\ C_p(T_1) = 4199$$

gives

$$C_p(52) = a_0 + a_1(52) = 4186$$

$$C_p(82) = a_0 + a_1(82) = 4199$$

Writing the equations in matrix form, we have

$$\begin{bmatrix} 1 & 52 \\ 1 & 82 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 4186 \\ 4199 \end{bmatrix}$$

Solving the above two equations gives

$$a_0 = 4163.5$$

$$a_1 = 0.43333$$

Hence

$$C_p(T) = a_0 + a_1 T$$

$$= 4163.5 + 0.43333T,\ \ 52 \le T \le 82$$

At $T = 61$,

$$C_p(61) = 4163.5 + 0.43333(61)$$

$$= 4189.9 \frac{\text{J}}{\text{kg} - °\text{C}}$$

Example 2

To find how much heat is required to bring a kettle of water to its boiling point, you are asked to calculate the specific heat of water at $61°\text{C}$. The specific heat of water is given as a function of time in Table 2.

Table 2 Specific heat of water as a function of temperature.

Temperature, T $(°\text{C})$	Specific heat, C_p $\left(\frac{\text{J}}{\text{kg} - °\text{C}}\right)$
22	4181
42	4179
52	4186
82	4199
100	4217

Determine the value of the specific heat at $T = 61°\text{C}$ using the direct method of interpolation and a second order polynomial. Find the absolute relative approximate error for the second order polynomial approximation.

Solution

For second order polynomial interpolation (also called quadratic interpolation), we choose the specific heat given by

$$C_p(T) = a_0 + a_1 T + a_2 T^2$$

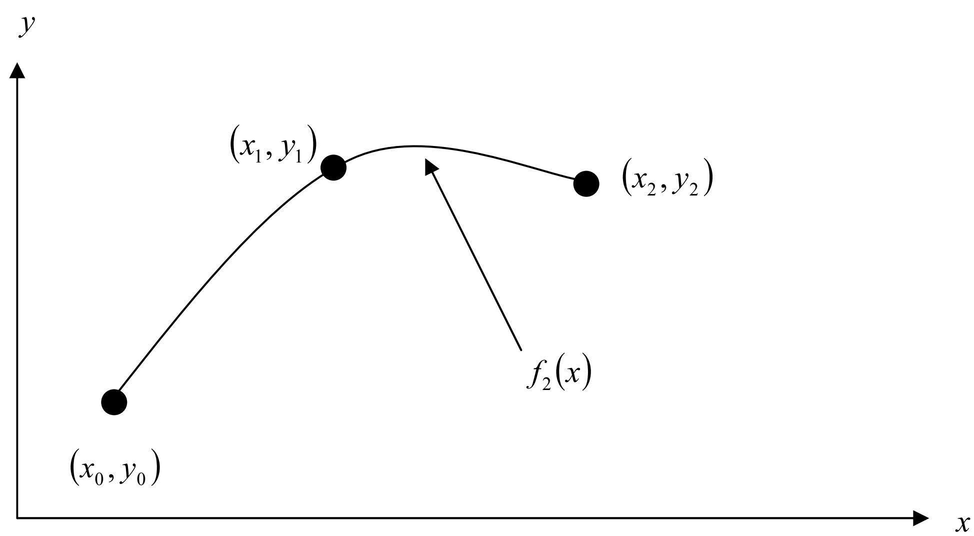

Figure 3 Quadratic interpolation.

Since we want to find the specific heat at $T = 61°\text{C}$, and we are using a second order polynomial, we need to choose the three data points that are closest to $T = 61°\text{C}$ that also bracket $T = 61°\text{C}$ to evaluate it. The three points are $T_0 = 42$, $T_1 = 52$, and $T_2 = 82$.

Then

$$T_0 = 42,\ C_p(T_0) = 4179$$

$$T_1 = 52,\ C_p(T_1) = 4186$$

$$T_2 = 82,\ C_p(T_2) = 4199$$

gives

$$C_p(42) = a_0 + a_1(42) + a_2(42)^2 = 4179$$

$$C_p(52) = a_0 + a_1(52) + a_2(52)^2 = 4186$$

$$C_p(82) = a_0 + a_1(82) + a_2(82)^2 = 4199$$

Writing the three equations in matrix form, we have

$$\begin{bmatrix} 1 & 42 & 1764 \\ 1 & 52 & 2704 \\ 1 & 82 & 6724 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 4179 \\ 4186 \\ 4199 \end{bmatrix}$$

Solving the above three equations gives

$$a_0 = 4135.0$$
$$a_1 = 1.3267$$
$$a_2 = -6.6667 \times 10^{-3}$$

Hence

$$C_p(T) = 4135.0 + 1.3267T - 6.6667 \times 10^{-3} T^2, \ \ 42 \le T \le 82$$

At $T = 61$,

$$C_p(61) = 4135.0 + 1.3267(61) - 6.6667 \times 10^{-3}(61)^2$$
$$= 4191.2 \frac{\text{J}}{\text{kg} - {}^\circ\text{C}}$$

The absolute relative approximate error $|\in_a|$ obtained between the results from the first and second order polynomial is

$$|\in_a| = \left| \frac{4191.2 - 4189.9}{4191.2} \right| \times 100$$
$$= 0.030063\%$$

Example 3

To find how much heat is required to bring a kettle of water to its boiling point, you are asked to calculate the specific heat of water at 61°C. The specific heat of water is given as a function of time in Table 3.

Table 3 Specific heat of water as a function of temperature.

Temperature, T $(^\circ\text{C})$	Specific heat, C_p $\left(\frac{\text{J}}{\text{kg} - {}^\circ\text{C}}\right)$
22	4181
42	4179
52	4186
82	4199
100	4217

Determine the value of the specific heat at $T = 61^\circ\text{C}$ using the direct method of interpolation and a third order polynomial. Find the absolute relative approximate error for the third order polynomial approximation.

Solution

For third order polynomial interpolation (also called cubic interpolation), we choose the specific heat given by

$$C_p(T) = a_0 + a_1 T + a_2 T^2 + a_3 T^3$$

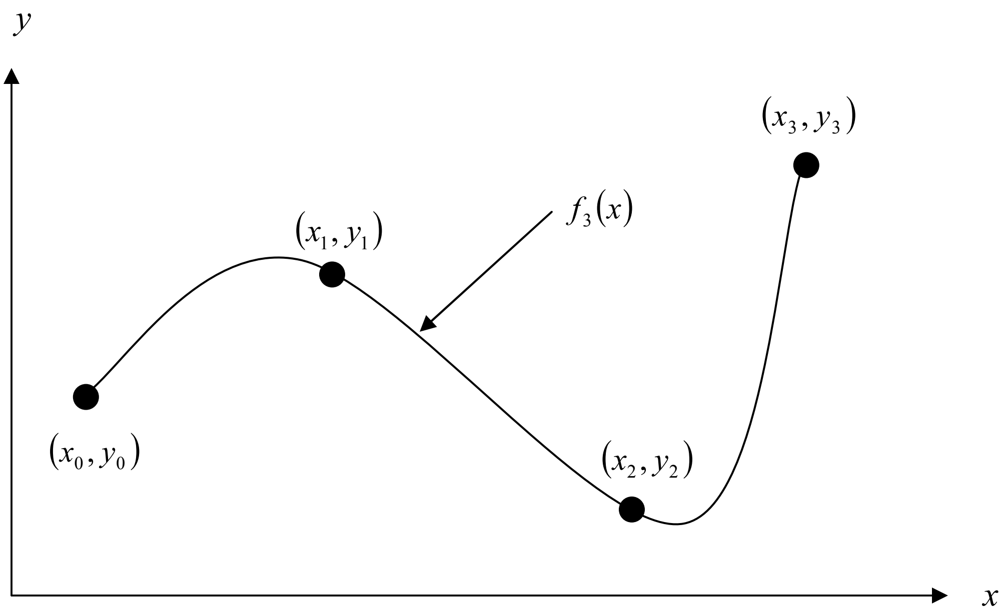

Figure 4 Cubic interpolation.

Since we want to find the specific heat at $T = 61°\text{C}$, and we are using a third order polynomial, we need to choose the four data points closest to $T = 61°\text{C}$ that also bracket $T = 61°\text{C}$ to evaluate it. The four points are $T_0 = 42,\ T_1 = 52,\ T_2 = 82$ and $T_3 = 100.$ (Choosing the four points as $T_0 = 22$, $T_1 = 42$, $T_2 = 52$ and $T_3 = 82$ is equally valid.)
Then

$$T_0 = 42, \quad C_p(T_0) = 4179$$
$$T_1 = 52, \quad C_p(T_1) = 4186$$
$$T_2 = 82, \quad C_p(T_2) = 4199$$
$$T_3 = 100, \quad C_p(T_3) = 4217$$

gives

$$C_p(42) = a_0 + a_1(42) + a_2(42)^2 + a_3(42)^3 = 4179$$
$$C_p(52) = a_0 + a_1(52) + a_2(52)^2 + a_3(52)^3 = 4186$$
$$C_p(82) = a_0 + a_1(82) + a_2(82)^2 + a_3(82)^3 = 4199$$
$$C_p(100) = a_0 + a_1(100) + a_2(100)^2 + a_3(100)^3 = 4217$$

Writing the four equations in matrix form, we have

$$\begin{bmatrix} 1 & 42 & 1764 & 7.4088\times10^4 \\ 1 & 52 & 2704 & 1.4061\times10^5 \\ 1 & 82 & 6724 & 5.5137\times10^5 \\ 1 & 100 & 10000 & 10^6 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 4179 \\ 4186 \\ 4199 \\ 4217 \end{bmatrix}$$

Solving the above four equations gives

$$a_0 = 4078.0$$
$$a_1 = 4.4771$$
$$a_2 = -0.062720$$
$$a_3 = 3.1849 \times 10^{-4}$$

Hence

$$C_p(T) = a_0 + a_1 T + a_2 T^2 + a_3 T^3$$
$$= 4078.0 + 4.4771T - 0.062720T^2 + 3.1849 \times 10^{-4} T^3, \quad 42 \le T \le 100$$
$$T(61) = 4078.0 + 4.4771(61) - 0.062720(61)^2 + 3.1849 \times 10^{-4}(61)^3$$
$$= 4190.0 \frac{\text{J}}{\text{kg} - {}^\circ\text{C}}$$

The absolute relative approximate error $|\in_a|$ obtained between the results from the second and third order polynomial is

$$|\in_a| = \left| \frac{4190.0 - 4191.2}{4190.0} \right| \times 100$$
$$= 0.027295\%$$

INTERPOLATION	
Topic	Direct Method of Interpolation
Summary	Examples of direct method of interpolation.
Major	Chemical Engineering
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Date	November 23, 2009
Web Site	http://numericalmethods.eng.usf.edu