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Note to the reader: This is a customized book for Old Dominion University. However, all available content is listed here in the table of contents, and the content that is not in the book is shown with a strikethrough (e.g. ~~Chapter 02.00B Physical problem—chemical engineering~~). If you are an instructor for a numerical methods course and you are interested in adopting this book, we can customize a book for you based on your syllabus. Please contact the first author at autarkaw@yahoo.com or 813-974-5626.

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- ~~What do you mean by vectors being linearly independent?~~

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 Prove that if a set of vectors are linearly independent, then a subset of the m vectors also has to be linearly independent.
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