Chapter 04.00A

Physical Problem for Simultaneous Linear Equations
General Engineering

Problem Statement
The upward velocity of a rocket is given at three different times in the following table

<table>
<thead>
<tr>
<th>Time, t</th>
<th>Velocity, v</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>106.8</td>
</tr>
<tr>
<td>8</td>
<td>177.2</td>
</tr>
<tr>
<td>12</td>
<td>279.2</td>
</tr>
</tbody>
</table>

The velocity data is approximated by a polynomial as
Figure 1 A rocket launched into space\textsuperscript{1}

\[ v(t) = at^2 + bt + c, \quad 5 \leq t \leq 12. \]

Set up the equations in matrix form to find the coefficients \( a, b, c \) of the velocity profile.

**Solution**

The polynomial is going through three data points \((t_1, v_1), (t_2, v_2), \) and \((t_3, v_3)\) where from the above table

\[
\begin{align*}
    t_1 &= 5, v_1 = 106.8 \\
    t_2 &= 8, v_2 = 177.2 \\
    t_3 &= 12, v_3 = 279.2
\end{align*}
\]

Requiring that \( v(t) = at^2 + bt + c \) passes through the three data points gives

\[
\begin{align*}
    v(t_1) &= v_1 = at_1^2 + bt_1 + c \\
    v(t_2) &= v_2 = at_2^2 + bt_2 + c \\
    v(t_3) &= v_3 = at_3^2 + bt_3 + c
\end{align*}
\]

Substituting the data \((t_1, v_1), (t_2, v_2), (t_3, v_3)\) gives

\[
\begin{align*}
    a(5^2) + b(5) + c &= 106.8 \\
    a(8^2) + b(8) + c &= 177.2 \\
    a(12^2) + b(12) + c &= 279.2
\end{align*}
\]

or

\[
\begin{align*}
    25a + 5b + c &= 106.8 \\
    64a + 8b + c &= 177.2 \\
    144a + 12b + c &= 279.2
\end{align*}
\]

This set of equations can be rewritten in the matrix form as

\[
\begin{bmatrix}
    25a + 5b + c \\
    64a + 8b + c \\
    144a + 12b + c
\end{bmatrix}
= \begin{bmatrix}
    106.8 \\
    177.2 \\
    279.2
\end{bmatrix}
\]

The above equation can be written as a linear combination as follows

\[
\begin{align*}
    a \begin{bmatrix} 25 \end{bmatrix} + b \begin{bmatrix} 5 \end{bmatrix} + c \begin{bmatrix} 1 \end{bmatrix} &= \begin{bmatrix} 106.8 \end{bmatrix} \\
    a \begin{bmatrix} 64 \end{bmatrix} + b \begin{bmatrix} 8 \end{bmatrix} + c \begin{bmatrix} 1 \end{bmatrix} &= \begin{bmatrix} 177.2 \end{bmatrix} \\
    a \begin{bmatrix} 144 \end{bmatrix} + b \begin{bmatrix} 12 \end{bmatrix} + c \begin{bmatrix} 1 \end{bmatrix} &= \begin{bmatrix} 279.2 \end{bmatrix}
\end{align*}
\]

\[\text{Source of rocket picture: NASA Langley Research Center, Office of Education, } \texttt{edu.larc.nasa.gov/pstp/}\]
and further using matrix multiplications gives
\[
\begin{bmatrix}
25 & 5 & 1 \\
64 & 8 & 1 \\
144 & 12 & 1
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c
\end{bmatrix}
=
\begin{bmatrix}
106.8 \\
177.2 \\
279.2
\end{bmatrix}
\]

The solution of the above three simultaneous linear equations will give the value of \(a, b, c\).

**QUESTIONS**

1. Solve for the values of \(a, b, c\).
2. Verify if you get back the value of the velocity data at \(t=5\) s.
3. Estimate the velocity of the rocket at \(t=7.5\) s?
4. Estimate the acceleration of the rocket at \(t=7.5\) s?
5. Estimate the distance covered by the rocket between \(t=5.5\) s and \(8.9\) s.
6. If the following data is given for the velocity of the rocket as a function of time, and you are asked to use a quadratic polynomial to approximate the velocity profile to find the velocity at \(t=16\) s, what data points would you choose and why?

<table>
<thead>
<tr>
<th>(t)</th>
<th>(v(t))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>227.04</td>
</tr>
<tr>
<td>15</td>
<td>362.78</td>
</tr>
<tr>
<td>20</td>
<td>517.35</td>
</tr>
<tr>
<td>22.5</td>
<td>602.97</td>
</tr>
<tr>
<td>30</td>
<td>901.67</td>
</tr>
</tbody>
</table>