Chapter 04.00A

Physical Problem for Simultaneous Linear Equations
General Engineering

Problem Statement

The upward velocity of a rocket is given at three different times in the following table

Time, t	Velocity, v
s	m/s
5	106.8
8	177.2
12	279.2

The velocity data is approximated by a polynomial as

Figure 1 A rocket launched into space[1]

$$v(t) = at^2 + bt + c, \quad 5 \le t \le 12.$$

Set up the equations in matrix form to find the coefficients a, b, c of the velocity profile.

Solution

The polynomial is going through three data points (t_1, v_1), (t_2, v_2), and (t_3, v_3) where from the above table

$$t_1 = 5, v_1 = 106.8$$
$$t_2 = 8, v_2 = 177.2$$
$$t_3 = 12, v_3 = 279.2$$

Requiring that $v(t) = at^2 + bt + c$ passes through the three data points gives

$$v(t_1) = v_1 = at_1^2 + bt_1 + c$$
$$v(t_2) = v_2 = at_2^2 + bt_2 + c$$
$$v(t_3) = v_3 = at_3^2 + bt_3 + c$$

Substituting the data (t_1, v_1), (t_2, v_2), (t_3, v_3) gives

$$a(5^2) + b(5) + c = 106.8$$
$$a(8^2) + b(8) + c = 177.2$$
$$a(12^2) + b(12) + c = 279.2$$

or

$$25a + 5b + c = 106.8$$
$$64a + 8b + c = 177.2$$
$$144a + 12b + c = 279.2$$

This set of equations can be rewritten in the matrix form as

$$\begin{bmatrix} 25a + & 5b + & c \\ 64a + & 8b + & c \\ 144a + & 12b + & c \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

The above equation can be written as a linear combination as follows

$$a\begin{bmatrix} 25 \\ 64 \\ 144 \end{bmatrix} + b\begin{bmatrix} 5 \\ 8 \\ 12 \end{bmatrix} + c\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

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[1] Source of rocket picture: NASA Langley Research Center, Office of Education, edu.larc.nasa.gov/pstp/

and further using matrix multiplications gives

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

The solution of the above three simultaneous linear equations will give the value of a, b, c.

QUESTIONS

1. Solve for the values of a, b, c.
2. Verify if you get back the value of the velocity data at t=5s.
3. Estimate the velocity of the rocket at t=7.5s?
4. Estimate the acceleration of the rocket at t=7.5 s?
5. Estimate the distance covered by the rocket between t=5.5 s and 8.9 s.
6. If the following data is given for the velocity of the rocket as a function of time, and you are asked to use a quadratic polynomial to approximate the velocity profile to find the velocity at t=16 s, what data points would you choose and why?

t	v(t)
s	m/s
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

SIMULTANEOUS LINEAR EQUATIONS

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