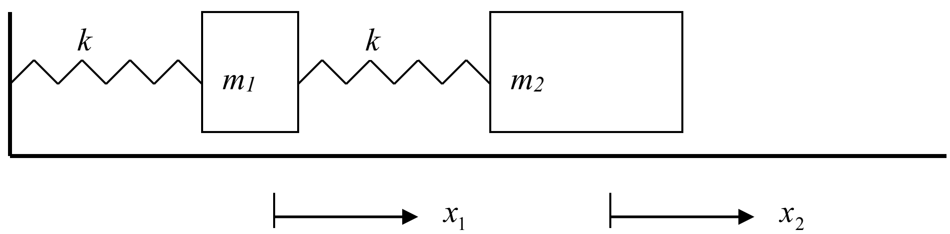
Chapter 04.00AA

Physical Problem for Matrix Algebra General Engineering

Problem Statement

Look at the spring-mass system as shown in the picture below.



Let $m_1 = 10, m_2 = 20, k = 15$, find the natural frequencies of the two masses.

Solution

Assume each of the two mass-displacements to be denoted by x_1 and x_2, and let us assume each spring has the same spring constant k. Then by applying Newton's 2nd and 3rd law of motion to develop a force-balance for each mass we have

$$m_1 \frac{d^2 x_1}{dt^2} = -kx_1 + k(x_2 - x_1)$$

$$m_2 \frac{d^2 x_2}{dt^2} = -k(x_2 - x_1)$$

Rewriting the equations, we have

$$m_1 \frac{d^2 x_1}{dt^2} - k(-2x_1 + x_2) = 0$$

$$m_2 \frac{d^2 x_2}{dt^2} - k(x_1 - x_2) = 0$$

Given $m_1 = 10, m_2 = 20, k = 15$

$$10\frac{d^2x_1}{dt^2} - 15(-2x_1 + x_2) = 0$$

$$20\frac{d^2x_2}{dt^2} - 15(x_1 - x_2) = 0$$

From vibration theory, the solutions can be of the form

$$x_i = A_i \sin(\omega t - \emptyset)$$

where

A_i = amplitude of the vibration of mass i,

ω = frequency of vibration,

$\emptyset$ = phase shift.

then

$$\frac{d^2x_i}{dt^2} = -A_i w^2 Sin(\omega t - \emptyset)$$

Substituting x_i and $\frac{d^2x_i}{dt^2}$ in equations,

$$-10A_1\omega^2 - 15(-2A_1 + A_2) = 0$$

$$-20A_2\omega^2 - 15(A_1 - A_2) = 0$$

gives

$$(-10\omega^2 + 30)A_1 - 15A_2 = 0$$

$$-15A_1 + (-20\omega^2 + 15)A_2 = 0$$

or

$$(-\omega^2 + 3)A_1 - 1.5A_2 = 0$$

$$-0.75A_1 + (-\omega^2 + 0.75)A_2 = 0$$

In matrix form, these equations can be rewritten as

$$\begin{bmatrix} -\omega^2 + 3 & -1.5 \\ -0.75 & -\omega^2 + 0.75 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -1.5 \\ -0.75 & 0.75 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} - \omega^2 \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Let $\omega^2 = \lambda$

$$[A] = \begin{bmatrix} 3 & -1.5 \\ -0.75 & 0.75 \end{bmatrix}$$

$$[X] = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$$

$$[A][X] - \lambda[X] = 0$$

$$[A][X] = \lambda[X]$$

In the above equation, λ is the eigenvalue and $[X]$ is the eigenvector corresponding to λ. As you can see, if we know λ for the above example we can calculate the natural frequency of the vibration

$$\omega = \sqrt{\lambda}$$

QUESTIONS

1. Why are the natural frequencies of vibration important?
2. Find the eigenvalues and eigenvectors for the problem.

SIMULTANEOUS LINEAR EQUATIONS	
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Summary	Spring-mass systems
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