03.01 Background of Quadratic Equations

Background

The exact solution to the cubic equation can be traced as far back as the late 15th century. In 1494, an Italian mathematician, named Luca Pacioli (ca. 1445-1509), looked at the cubic equation and expressed the belief that such a solution was impossible (Kline, 1972, p. 237). This observation not only acted as a challenge to the Italian mathematical community, but also set the stage for the sixteenth-century Italian algebraists and their quest for the solution of the cubic (Dunham, 1990, p. 134).

The first mathematician to take Pacioli's challenge was a professor of mathematics at University of Bologna, named Scipione dal Ferro (1465-1526). Around 1500, dal Ferro discovered a formula that solved the so-called "depressed cubic." This is a third-degree equation that lacks its second-degree, or quadratic, term (Dunham, 1990, p. 134). The following is an example of a depressed cubic.

$ax^3 + cx + d = 0.$

Although dal Ferro made a significant discovery in the quest to solve the cubic, he never published his method! The cubic's solution remained an absolute secret until about 1510, when dal Ferro confided his method to Antonio Maria Fior and his son-in-law and successor, Annibale della Nave (Kline, 1972, p. 263).

To understand why dal Ferro never told anyone of his achievement, one has to consider the nature of the Renaissance university at that time. There, academic appointments were by no means secure. At any time and from anyone, public challenges could be issued to mathematicians like dal Ferro. If they were not constantly prepared to do scholarly battle with challengers, their reputations and careers could be jeopardized (Dunham, 1990, p. 135).

Thus, a major new discovery was a powerful weapon. As William Dunham points out in his book Journey Through Genius (1990), dal Ferro would have an ace-in-the-hole if someone challenged him:

"Should an opponent appear with a list of problems to be solved, dal Ferro could counter with a list of depressed cubics. Even if dal Ferro were stumped by some of his challenger's problems, he could feel confident that his cubics, baffling to all but himself, should guarantee the downfall of his unfortunate adversary (p.135)".

The next person to enter the scene was Niccolo Fontana Tartaglia of Brescia (1499-1557). In 1535 Antonio Maria Fior challenged Tartaglia to solve thirty "depressed cubics", thereby placing Tartaglia in a bind. However, after struggling with the problem all the way up to the deadline of the challenge, Tartaglia discovered the solution to the "depressed cubic". In a great public triumph, Tartaglia prevailed brilliantly over the less gifted challenger, Fior (Dunham, 1990, p. 135).

The last main figure in the quest for the solution to the cubic equation was Girolamo Cardano (1501-1576). Cardano had heard of the challenge between Fior and Tartaglia and desired to learn more about Tartaglia's technique for solving cubic equations. Cardano was so impressed with Tartaglia's work that he boldly insisted Tartaglia to divulge the secret behind the cubic equation. After many letters to Tartaglia, on March 25, 1539, Tartaglia

revealed the secret of the "depressed cubic" in an obscure verse form after a pledge from Cardano to keep it secret.

Finally in 1543, Cardano and his pupil Lodovico Ferrari (1522-1565) were inspecting the papers of Scipione dal Ferro and discovered that dal Ferro's method for solving the "depressed" cubic was the same as Tartaglia's (Dunham, 1990, p. 141). With this discovery, Cardano realized that now he could publish the solution without breaking his promise to Tartaglia. Instead of using Tartaglia's solution, he would give credit to dal Ferro for the solution to the "depressed" cubic.

Therefore, in the year 1545, Cardano published the solution to the "depressed cubic" equation in his Ars Magna. In Chapter 11 of his Ars Magna, Cardano gave credit where credit was due for the solution:

"Scipio Ferro of Bologna well-nigh thirty years ago discovered this rule and handed it on to Antonio Maria Fior of Venice whose contest with Niccolo Tartaglia of Brescia gave Niccolo occasion to discover it." (Dunham, 1990, p. 141)

Hence, this is how the solution to the cubic equation came about.

Note: Do you want to see the derivation and example of how to find the exact solution of a cubic equation?.

References:

Dunham, W. (1990). Journey through genius: the great theorems of mathematics. New York: Wiley and Sons, Inc.

Kline, M. (1972). Mathematical thought from ancient to modern times. New York: Oxford University Press.

The "cubic formula". Available:

http://www.groups.dcs.stand.ac.uk/history/HistTopics/Quadratic_etc_equations.html#51 [September 4, 2000].

Recommended Reading:

Boyer, C. B. (1991) A history of mathematics: 2nd edition. New York: Wiley and Sons, Inc.

Dunham, W. (1994). The mathematical universe: an alphabetical journey through the great proofs, problems, and personalities. New York: Wiley and Sons, Inc.

NONLINEAR EQUATIONS	
Topic	Background of Quadratic Equations
Summary	Textbook notes on the background of solving cubic equations.
Major	All Majors of Engineering
Authors	Clayton Rabens, Aaron Cline and Autar Kaw
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