

Chapter 03.00E

Physical Problem for Nonlinear Equations Electrical Engineering

Summary

Thermistors are temperature measuring devices based on that resistance of materials changes with temperature. To find whether the resistor is calibrated properly, one needs to solve a problem of a nonlinear equation.

Thermistors are temperature-measuring devices based on the principle that the thermistor material exhibits a change in electrical resistance with a change in temperature. By measuring the resistance of the thermistor material, one can then determine the temperature.

Thermistors are generally a piece of semiconductor (Figure 1) made from metal oxides such as those of manganese, nickel, cobalt, etc. These pieces may be made into a bead, disk, wafer, etc depending on the application.

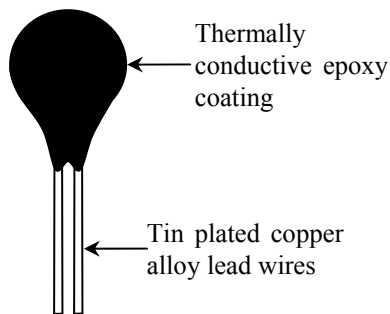


Figure 1. Sketch of a thermistor.

There are two types of thermistors – negative temperature coefficient (NTC) and positive temperature coefficient (PTC) thermistors. For NTCs, the resistance decreases with temperature, while for PTCs, the temperature increases with temperature. It is the NTCs that are generally used for temperature measurement.

Why would we want to use thermistors for measuring temperature as opposed to other choices such as thermocouples? It is because thermistors have

- high sensitivity giving more accuracy,

- a fast response to temperature changes for accuracy and quicker measurements, and
- relatively high resistance for decreasing the errors caused by the resistance of lead wires themselves.

But thermistors have a nonlinear output and are valued for a limited range. So, when a thermistor is manufactured, the manufacturer supplies a resistance vs. temperature curve. The curve generally used that gives an accurate representation is given by Steinhart and Hart equation

$$\frac{1}{T} = a_0 + a_1 \ln(R) + a_3 \{\ln(R)\}^3 \quad (1)$$

where

T is temperature in Kelvin, and

R is resistance in ohms.

a_0, a_1, a_3 are constants of the calibration curve.

As an example, for an actual thermistor – Part No 10K3A made by Betatherm sensors, the values of the three coefficients are given as

$$a_0 = 1.129241 \times 10^{-4}$$

$$a_1 = 2.341077 \times 10^{-4}$$

$$a_3 = 8.775468 \times 10^{-8}$$

and are found by measuring the resistance of the thermistor at three reference points (namely 0°C , 25°C and 70°C in this case) and using equation (1) to set up three simultaneous linear equations to find the three constants a_0, a_1, a_3 . The resulting Steinhart-Hart equation for the 10K3A Betatherm thermistor is

$$\frac{1}{T} = 1.129241 \times 10^{-3} + 2.341077 \times 10^{-4} \ln(R) + 8.775468 \times 10^{-8} \{\ln(R)\}^3 \quad (2)$$

where note that T is in Kelvin and R is in ohms.

Using a digital system to measure temperature, an analog system is used to measure the thermistor resistances and convert that to a temperature reading. You want to confirm that the Resistance vs. Temperature data compares well with the published R/T data for the range for which the thermistor will be used. For example for the above thermistor, error of no more than $\pm 0.01^\circ\text{C}$ is acceptable. What is the range of the resistance then you can consider to be within this acceptable limit at 19°C ? To find this we need to solve the equation for a temperature of $19 \pm 0.01 = 18.99^\circ\text{C}$ to 19.01°C range. These equations are

$$\frac{1}{19.01 + 273.15} = 1.129241 \times 10^{-3} + 2.341077 \times 10^{-4} \ln(R) + 8.775468 \times 10^{-8} \{\ln(R)\}^3$$

$$3.42278 \times 10^{-3} = 1.129241 \times 10^{-3} + 2.341077 \times 10^{-4} \ln(R) + 8.775468 \times 10^{-8} \{\ln(R)\}^3 \quad (3)$$

and

$$\frac{1}{18.99 + 273.15} = 1.129241 \times 10^{-3} + 2.341077 \times 10^{-4} \ln(R) + 8.775468 \times 10^{-8} \{\ln(R)\}^3$$

$$3.42301 \times 10^{-3} = 1.129241 \times 10^{-3} + 2.341077 \times 10^{-4} \ln(R) + 8.775468 \times 10^{-8} \{\ln(R)\}^3 \quad (4)$$

Equations (3) and (4) are independent nonlinear equations that need to be solved for R .

References

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5. Steinhart, J.S. and Hart, S.R., 1968. "Calibration Curves for Thermistors," Deep Sea Research 15:497.
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7. Siwek, W.R., et al. 1992. "A Precision Temperature Standard Based on the Exactness of Fit of Thermistor Resistance-Temperature Data Using Third Degree Polynomials," Temperature, Its Measurement and Control in Science and Industry, Vol. 6, James F. Schooley, ed., American Institute of Physics, New York, NY:491-496.

Questions

Answer the following questions

1. Note that if we substitute $x = \ln(R)$, the equation becomes a cubic equation in x . The cubic equation will have three roots. Could some of these roots be complex? If so how many?
2. Solving the cubic equation exactly would require major effort. However using numerical techniques, we can solve this equation and any other equation of the form $f(x) = 0$. Solve the above equation by all the methods you have learned assuming you want at least 3 significant digits to be correct in your answer.
3. How can you use the knowledge of the physics of the problem to develop initial guess(es) for the numerical methods?
4. If more than one root of the above equation is real, how do you choose the valid root? Or, are all the possible real roots physically acceptable?

Topic	NONLINEAR EQUATION
Sub Topic	Physical Problem
Summary	
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Last Revised	December 23, 2009
Web Site	http://numericalmethods.eng.usf.edu