Multiple-Choice Test
Informal Development of Fast Fourier Transform (FFT)
Chapter 11.05
COMPLETE SOLUTION SET

1. Using the definition $E = e^{-i\frac{2\pi}{N}}$, and the Euler identity $e^{\pm i\theta} = \cos(\theta) \pm i\sin(\theta)$, the value of $E^{\frac{N}{6}}$ can be computed as

   (A) $0.866 - 0.5i$
   (B) $-0.866 + 0.5i$
   (C) $-0.5 - 0.866i$
   (D) $0.5 - 0.866i$

Solution

The correct answer is (D).

$$E^{\left(\frac{N}{6}\right)} = e^{\left(\frac{-i\times 2\pi}{N}\times\frac{N}{6}\right)}$$

$$= e^{\left(-i\times\frac{\pi}{3}\right)}$$

$$= Cos\left(\frac{-\pi}{3}\right) + i \times Sin\left(\frac{-\pi}{3}\right)$$

$$= 0.5000 - i \times 0.866$$

2. Using the definition $E = e^{-i\frac{2\pi}{N}}$, and the Euler identity $e^{\pm i\theta} = \cos(\theta) \pm i\sin(\theta)$, the value of E^{6N} can be computed as

(A) $1+i$
(B) $1-i$
(C) 1
(D) -1

Solution
The correct answer is (C).

$$E^{(6\times N)} = e^{\left(\frac{-i\times 2\pi}{N}\times 6N\right)}$$

$$= e^{(-i\times 12\times\pi)}$$

$$= Cos(-12\times\pi) + i\times Sin(-12\times\pi)$$

$$= 1 + i\times(0)$$

$$= 1$$

3. Given $N = 2$, and $\{f\} = \begin{Bmatrix} f(0) \\ f(1) \end{Bmatrix} = \begin{Bmatrix} 14+6i \\ -2+4i \end{Bmatrix}$. The first part of $\widetilde{C}_n = \widetilde{C}(n) = \sum_{k=0}^{N-1} f(k)E^{nk}$ can be expressed as

$$\widetilde{C}(0) = \sum_{k=0}^{1} f(k)E^{nk} = f(0)E^{(0)(0)} + f(1)E^{(0)(1)}$$

$$\widetilde{C}(1) = f(0)E^{(1)(0)} + f(1)E^{(1)(1)}$$

The values for $\begin{Bmatrix} \widetilde{C}(0) \\ \widetilde{C}(1) \end{Bmatrix}$ can be computed as

(A) $\begin{Bmatrix} 12+10i \\ 16+2i \end{Bmatrix}$

(B) $\begin{Bmatrix} 10+12i \\ 2+16i \end{Bmatrix}$

(C) $\begin{Bmatrix} -12+10i \\ -16+2i \end{Bmatrix}$

(D) $\begin{Bmatrix} 10-12i \\ 2-16i \end{Bmatrix}$

Solution

The correct answer is (A).

Based on Eq.(5) and with $N = 2$, one has:

$$\widetilde{C}_0 = f(0)\times E^{(0\times 0)} + f(1)\times E^{(0\times 1)}$$

$$= f(0)\times 1.0 + f(1)\times 1.0$$

$$= (14+6i) + (-2+4i)$$

$$\widetilde{C}_0 = 12+10i$$

$$\widetilde{C}_1 = f(0)\times E^{(1\times 0)} + f(1)\times E^{(1\times 1)}$$

$$\widetilde{C}_1 = f(0)\times 1.0 + f(1)\times E$$

Since,

$$E = e^{\left(-i \times 2 \times \frac{\pi}{N}\right)} = e^{(-i \times \pi)}$$

$$E = Cos(\pi) - i \times Sin(\pi)$$

$$E = -1.0 - i \times 0.0$$

$$E = -1.0$$

Hence,

$$\widetilde{C}_1 = (14 + 6i) + (-2 + 4i) \times (-1.0)$$

$$\widetilde{C}_1 = (14 + 6i) + (2 - 4i)$$

$$\widetilde{C}_1 = (16 + 2i)$$

Thus, the correct choice should be (A).

4. For $N = 2^4 = 16$, level $L = 2$ and referring to the figure shown on next page, the only terms of vector $f_2(-)$ which only need to compute are:

(A) $f_2(4-7, 12-15)$
(B) $f_2(0-3, 8-11)$
(C) $f_2(0-7)$
(D) $f_2(8-15)$

Solution
The correct answer is (B).

For level $L=2$, the 2 "companion" nodes are separated by the "distance":

$$\text{distance} = \frac{N}{2^L} = \frac{16}{2^2} = \frac{16}{4} = 4$$

Thus, at Level $L=2$, one will compute the first 4 components, then skip the next 4, then compute the next 4, then skip the next 4.

In other words, one will

Compute $f_2(0), f_2(1), f_2(2), f_2(3),$

Skip $f_2(4), f_2(5), f_2(6), f_2(7)$

Compute $f_2(8), f_2(9), f_2(10), f_2(11)$

Skip $f_2(12), f_2(13), f_2(14), f_2(15)$

5. For $N = 2^4 = 16$, level $L = 3$ and referring to referring to the figure shown on next page, the only companion nodes associated with $f_3(0)$ and $f_3(1)$ are

(A) $f_3(4)$ and $f_3(5)$

(B) $f_3(6)$ and $f_3(7)$

(C) $f_3(14)$ and $f_3(15)$

(D) $f_3(2)$ and $f_3(3)$

Solution

The correct answer is (D).

For $N = 16$, and Level $L = 3;$ then

"distance" (between any 2 companion nodes) $= \frac{N}{2^L} = \frac{16}{2^3} = \frac{16}{8} = 2$

Hence, the companion node of $f_3(0)\ is\ f_3(0 + distance) = f_3(2)$

the companion node of $f_3(1)\ is\ f_3(1 + distance) = f_3(3)$

6. Given $N = 4$, and $f_0\begin{pmatrix}0\\1\\2\\3\end{pmatrix} = \begin{Bmatrix}-4+i\\1-2i\\-2+3i\\3-4i\end{Bmatrix}$. Corresponding to level $L = 1$, one can compute $f_1(2)$ as

(A) $-2-2i$
(B) $4-6i$
(C) $4-6i$
(D) $-4-4i$

Solution
The correct answer is (A).

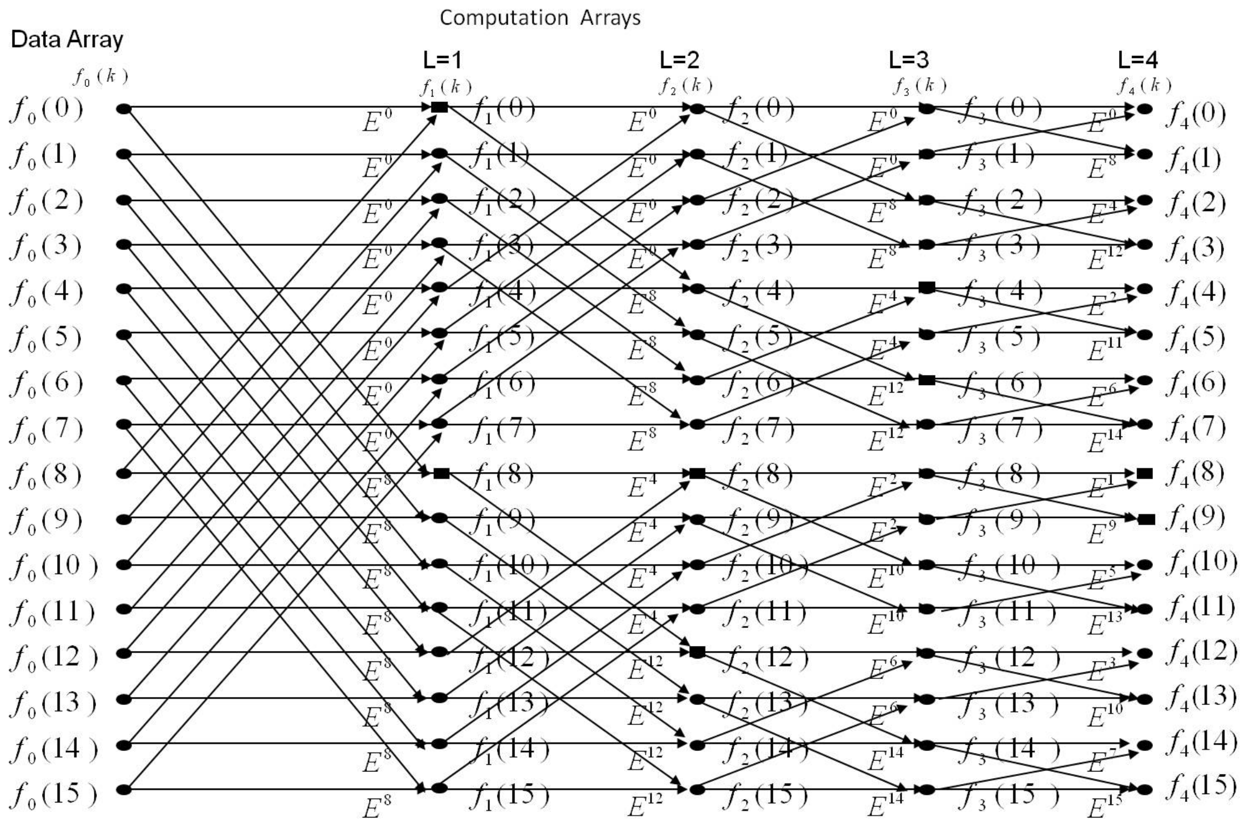
Using Eq.(11c), one obtains:

$$f_1(2) = f(0) - E^{(0)} \times f(2)$$

$$f_1(2) = (-4+i) - 1.0 \times (-2+3i)$$

$$f_1(2) = (-4+i) + (2-3i)$$

$$f_1(2) = -2-2i$$



Computation Arrays
Data Array
L=1
L=2
L=3
L=4