Multiple-Choice Test

Chapter 08.04
Runge-Kutta 4th Order Method

1. To solve the ordinary differential equation
   \[ \frac{dy}{dx} + xy^2 = \sin x, y(0) = 5, \]
   by Runge-Kutta 4th order method, you need to rewrite the equation as
   (A) \( \frac{dy}{dx} = \sin x - xy^2, y(0) = 5 \)
   (B) \( \frac{dy}{dx} = \frac{1}{3} (\sin x - xy^2), y(0) = 5 \)
   (C) \( \frac{dy}{dx} = \frac{1}{3} (-\cos x - \frac{xy^3}{3}), y(0) = 5 \)
   (D) \( \frac{dy}{dx} = \frac{1}{3} \sin x, y(0) = 5 \)

2. Given \( \frac{dy}{dx} + 5y^2 = \sin x, y(0.3) = 5 \) and using a step size of \( h = 0.3 \), the value of \( y(0.9) \) using Runge-Kutta 4th order method is most nearly
   (A) \(-0.25011 \times 10^{-6}\)
   (B) \(-4297.4\)
   (C) \(-1261.5\)
   (D) \(0.88498\)

3. Given \( \frac{dy}{dx} + y^2 = e^x, y(0.3) = 5 \), and using a step size of \( h = 0.3 \), the best estimate of \( \frac{dy}{dx}(0.9) \) Runge-Kutta 4th order method is most nearly
   (A) \(-1.6604\)
   (B) \(-1.1785\)
   (C) \(-0.45831\)
   (D) \(2.7270\)
4. The velocity (m/s) of a parachutist is given as a function of time (seconds) by
\[ v(t) = 55.8 \tanh(0.17t), \quad t \geq 0 \]
Using Runge-Kutta 4th order method with a step size of 5 seconds, the distance in meters traveled by the body from \( t = 2 \) to \( t = 12 \) seconds is estimated most nearly as

(A) 341.43
(B) 428.97
(C) 429.05
(D) 703.50

5. Runge-Kutta method can be derived from using first three terms of Taylor series of writing the value of \( y_{i+1} \), that is the value of \( y \) at \( x_{i+1} \), in terms of \( y_i \) and all the derivatives of \( y \) at \( x_i \). If \( h = x_{i+1} - x_i \), the explicit expression for \( y_{i+1} \) if the first five terms of the Taylor series are chosen for the ordinary differential equation
\[ \frac{dy}{dx} + 5y = 3e^{-2x}, \quad y(0) = 7, \]
would be

(A) \[ y_{i+1} = y_i + (3e^{-2x_i} - 5y_i)h + \frac{5h^2}{2} \]
(B) \[ y_{i+1} = y_i + (3e^{-2x_i} - 5y_i)h + \left( -21e^{-2x_i} + 25y_i \right) \frac{h^2}{2} + \left( -483e^{-2x_i} + 625y_i \right) \frac{h^3}{6} + \left( -30090e^{-2x_i} + 390625y_i \right) \frac{h^4}{24} \]
(C) \[ y_{i+1} = y_i + (3e^{-2x_i} - 5y_i)h + \left( -6e^{-2x_i} \right) \frac{h^2}{2} + \left( 12e^{-2x_i} \right) \frac{h^3}{6} + \left( -24e^{-2x_i} \right) \frac{h^4}{24} \]
(D) \[ y_{i+1} = y_i + (3e^{-2x_i} - 5y_i)h + \left( -6e^{-2x_i} + 5 \right) \frac{h^2}{2} + \left( 12e^{-2x_i} \right) \frac{h^3}{6} + \left( -24e^{-2x_i} \right) \frac{h^4}{24} \]
A hot solid cylinder is immersed in a cool oil bath as part of a quenching process. This process makes the temperature of the cylinder, $\theta_c$, and the bath, $\theta_b$, change with time. If the initial temperature of the bar and the oil bath is given as 600° C and 27°C, respectively, and

- Length of cylinder = 30 cm
- Radius of cylinder = 3 cm
- Density of cylinder = 2700 kg/m$^3$
- Specific heat of cylinder = 895 J/kg $\cdot$ K
- Convection heat transfer coefficient = 100 W/m$^2$ $\cdot$ K
- Specific heat of oil = 1910 J/kg $\cdot$ K
- Mass of oil = 2 kg

the coupled ordinary differential equation giving the heat transfer are given by

\[
\begin{align*}
(A) & \quad 362.4 \frac{d\theta_c}{dt} + \theta_c = \theta_b \\
(B) & \quad 675.5 \frac{d\theta_b}{dt} + \theta_b = \theta \\
(C) & \quad 362.4 \frac{d\theta_c}{dt} - \theta_c = \theta_b \\
(D) & \quad 675.5 \frac{d\theta_b}{dt} - \theta_b = \theta_c 
\end{align*}
\]

For a complete solution, refer to the links at the end of the book.