Holistic Numerical Methods Institute
committed to bringing numerical methods to undergraduates

Multiple-Choice Test
Trapezoidal Rule
Integration
COMPLETE SOLUTION SET

1. The two-segment trapezoidal rule of integration is exact for integrating at most ________ order polynomials.

   (A) first

   (B) second

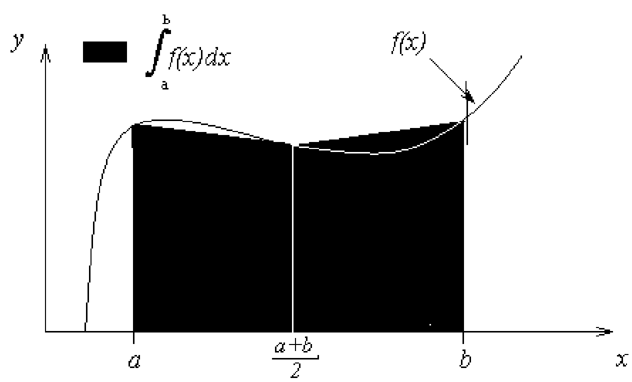
   (C) third

   (D) fourth

Solution

The correct answer is (A).

The single segment trapezoidal rule is exact for at most a first order polynomial. The two segment trapezoidal rule is also exact only for the same order of polynomial, that is, a first order polynomial.



2. The value of $\int_{0.2}^{2.2} xe^x dx$ by the using one-segment trapezoidal rule is most nearly

(A) 11.672

(B) 11.807

(C) 20.099

(D) 24.119

Solution

The correct answer is (C).

$$\int_a^b f(x)dx \approx (b-a)\left[\frac{f(a)+f(b)}{2}\right]$$

where

$$a = 0.2$$

$$b = 0.2$$

$$f(x) = xe^x$$

$$f(0.2) = 0.2e^{0.2}$$
$$= 0.24428$$

$$f(2.2) = 2.2e^{2.2}$$
$$= 19.855$$

$$\int_{0.2}^{2.2} xe^x dx \approx (2.2-0.2)\left[\frac{0.24428+19.855}{2}\right]$$
$$= 2 \times 10.050$$
$$= 20.099$$

3. The value of $\int_{0.2}^{2.2} xe^x dx$ by using the three-segment trapezoidal rule is most nearly

(A) 11.672

(B) 11.807

(C) 12.811

(D) 14.633

Solution

The correct answer is (C).

$$\int_a^b f(x)dx \approx \frac{b-a}{2n}\left[f(a)+2\left\{\sum_{i=1}^{n-1} f(a+ih)\right\}+f(b)\right]$$

where

$$a = 0.2$$
$$b = 2.2$$
$$n = 3$$
$$h = \frac{b-a}{n}$$
$$= \frac{2.2-0.2}{3}$$
$$= 0.66667$$
$$f(x) = xe^x$$

Thus

$$\int_a^b f(x)dx \approx \frac{2.2-0.2}{2\times 3}\left[f(0.2)+2\left\{\sum_{i=1}^{3-1} f(0.2+i\times 0.66667)\right\}+f(2.2)\right]$$
$$= \frac{2}{6}\left[f(0.2)+2\left\{\sum_{i=1}^{2} f(0.2+i\times 0.66667)\right\}+f(2.2)\right]$$
$$= \frac{1}{3}\left[f(0.2)+2f(0.86667)+2f(1.5333)+f(2.2)\right]$$

where

$$f(0.2) = 0.2e^{0.2}$$
$$= 0.24428$$
$$f(0.86667) = 0.86667e^{0.86667}$$
$$= 2.0618$$
$$f(1.5333) = 1.5333e^{1.5333}$$
$$= 7.1048$$
$$f(2.2) = 2.2e^{2.2}$$
$$= 19.855$$

Hence

$$\int_{0.2}^{2.2} xe^x dx \approx 0.33333[0.24428 + 2 \times 2.0618 + 2 \times 7.1048 + 19.855]$$
$$= 0.33333[38.433]$$
$$= 12.811$$

4. The velocity of a body is given by

$$v(t) = 2t, \qquad 1 \le t \le 5$$
$$= 5t^2 + 3, \quad 5 < t \le 14$$

where t is given in seconds, and v is given in m/s. Use the two-segment trapezoidal rule to find the distance covered by the body from $t = 2$ to $t = 9$ seconds.

(A) 935.0 m

(B) 1039.7 m

(C) 1260.9 m

(D) 5048.9 m

Solution

The correct answer is (C).

$$\int_a^b v(t)dt \approx \frac{b-a}{2n}\left[v(a) + 2\left\{\sum_{i=1}^{n-1} v(a+ih)\right\} + v(b)\right]$$

where

$$a = 2$$
$$b = 9$$
$$n = 2$$
$$h = \frac{b-a}{n}$$
$$= \frac{9-2}{2}$$
$$= 3.5$$
$$v(t) = 2t, \qquad 1 \le t \le 5$$
$$= 5t^2 + 3, \quad 5 < t \le 14$$

Thus

$$\int_2^9 v(t)dt \approx \frac{9-2}{2\times 2}\left[v(2) + 2\left\{\sum_{i=1}^{2-1} v(2 + i \times 3.5)\right\} + v(9)\right]$$
$$= \frac{7}{4}\left[v(2) + 2v(5.5) + v(9)\right]$$
$$v(2) = 2\times 2$$
$$= 4 \text{ m/s}$$
$$v(5.5) = 5\times 5.5^2 + 3$$
$$= 154.25 \text{ m/s}$$
$$v(9) = 5\times 9^2 + 3$$
$$= 408 \text{ m/s}$$

$$\int_2^9 v(t)dt \approx 1.75[4 + 2 \times 154.25 + 408]$$
$$= 1.75[720.5]$$
$$= 1260.9 \text{ m}$$

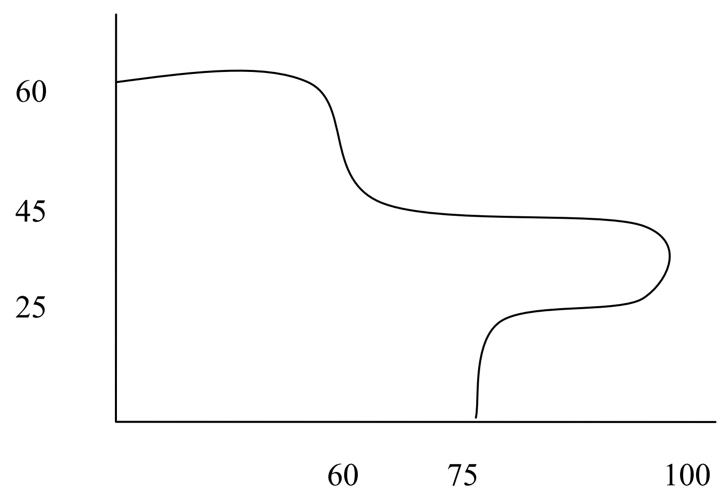
5. The shaded area shows a plot of land available for sale. Your best estimate of the area of the land is most nearly

(A) 2500 m^2

(B) 4775 m^2

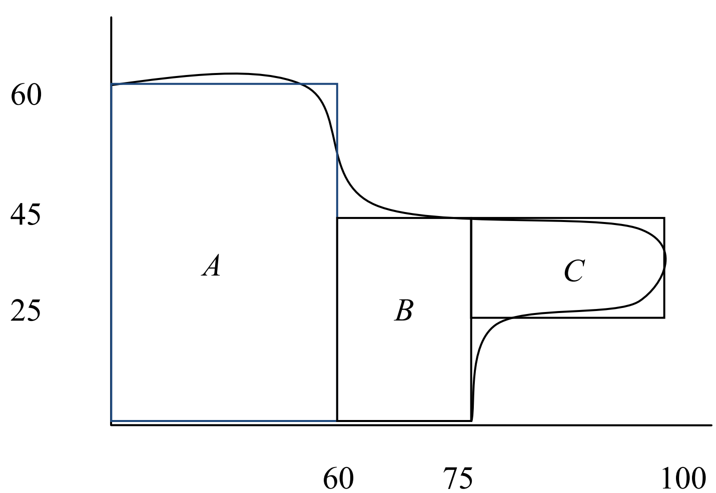
(C) 5250 m^2

(D) 6000 m^2



Solution

The correct answer is (B).



$$
\begin{aligned}
A &= 60 \times 60 \\
&= 3600\,\mathrm{m}^2 \\
B &= 45 \times 15 \\
&= 675\,\mathrm{m}^2 \\
C &= 20 \times 25 \\
&= 500\,\mathrm{m}^2 \\
Area &\approx A + B + C \\
&= 3600 + 675 + 500 \\
&= 4775\,\mathrm{m}^2
\end{aligned}
$$

6. The following data of the velocity of a body is given as a function of time.

Time (s)	0	15	18	22	24
Velocity (m/s)	22	24	37	25	123

The distance in meters covered by the body from $t = 12\,\text{s}$ to $t = 18\,\text{s}$ calculated using the trapezoidal rule with unequal segments is

(A) 162.90

(B) 166.00

(C) 181.70

(D) 436.50

Solution

The correct answer is (A).

Use the trapezoidal rule with unequal segments.

$$\int_{12}^{18} v(t)dt = \int_{12}^{15} v(t)dt + \int_{15}^{18} v(t)dt$$

$$v(15) = 24\,\text{m/s}$$

$$v(18) = 37\,\text{m/s}$$

To find the value of the velocity at 12 s, we will use linear interpolation.

$$v(t) = a_0 + a_1 t,\ 0 \le t \le 15$$

At $t = 0\,\text{s}$

$$22 = a_0 + a_1 0$$

At $t = 15\,\text{s}$

$$24 = a_0 + a_1 15$$

which gives

$$a_0 = 22$$

$$a_1 = 0.13333$$

Hence,

$$v(t) = 22 + 0.13333t,\ 0 \le t \le 15$$

$$v(12) = 22 + 0.13333 \times 12$$

$$= 23.600\,\text{m/s}$$

$$\int_{12}^{18} v(t)dt \approx (15-12)\left[\frac{v(12)+v(15)}{2}\right] + (18-15)\left[\frac{v(15)+v(18)}{2}\right]$$

$$= (15-12)\left[\frac{23.6+24}{2}\right] + (18-15)\left[\frac{24+37}{2}\right]$$

$$= 162.90\,\text{m}$$