Holistic Numerical Methods Institute
committed to bringing numerical methods to undergraduates

Multiple-Choice Test
Simpson's 1/3 Rule
Integration
COMPLETE SOLUTION SET

1. The highest order of polynomial integrand for which Simpson's 1/3 rule of integration is exact is

(A) first
(B) second
(C) third
(D) fourth

Solution
The correct answer is (C).

Simpson's 1/3 rule of integration is exact for integrating polynomials of third order or less. Although Simpson's 1/3 rule is derived by approximating the integrand by a second order polynomial, the area under the curve is exact for a third order polynomial. Without proof it can be shown that the truncation error in Simpson's 1/3 rule is $E_t = -\frac{(b-a)^5}{2880} f^{(4)}(\zeta), \quad a < \zeta < b$.

Since the fourth derivative of a third order polynomial is zero, the truncation error would be zero. Hence Simpson's 1/3 rule is exact for integrating polynomials of third order or less.

2. The value of $\int_{0.2}^{2.2} e^x dx$ by using 2-segment Simpson's 1/3 rule most nearly is

(A) 7.8036
(B) 7.8423
(C) 8.4433
(D) 10.246

Solution

The correct answer is (B).

The multiple segment equation for Simpson's 1/3 rule is

$$\int_a^b f(x)dx \approx \frac{b-a}{3n}\left[f(x_0) + 4\sum_{\substack{i=1 \\ i=\text{odd}}}^{n-1} f(x_i) + 2\sum_{\substack{i=2 \\ i=\text{even}}}^{n-2} f(x_i) + f(x_n)\right]$$

Using two-segments gives

$$a = 0.2$$
$$b = 2.2$$
$$n = 2$$
$$h = \frac{b-a}{n}$$
$$= \frac{2.2-0.2}{2}$$
$$= 1$$
$$x_0 = 0.2$$
$$x_1 = x_0 + h$$
$$= 0.2 + 1$$
$$= 1.2$$
$$x_2 = 1.2 + 1$$
$$= 2.2$$

$$\int_{0.2}^{2.2} e^x dx \approx \frac{2.2-0.2}{3\times 2}\left[f(0.2)+4\sum_{\substack{i=1\\ i=\text{odd}}}^{2-1} f(x_i)+2\sum_{\substack{i=2\\ i=\text{even}}}^{2-2} f(x_i)+f(2.2)\right]$$

$$=\frac{2.2-0.2}{3\times 2}\left[f(0.2)+4\sum_{\substack{i=1\\ i=\text{odd}}}^{1} f(x_i)+2\sum_{\substack{i=2\\ i=\text{even}}}^{0} f(x_i)+f(2.2)\right]$$

$$=\frac{2.2-0.2}{3\times 2}\left[f(0.2)+4f(1.2)+f(2.2)\right]$$

$$=0.33333\left[e^{0.2}+4\times e^{1.2}+e^{2.2}\right]$$

$$=0.33333\left[23.527\right]$$

$$=7.8423$$

3. The value of $\int_{0.2}^{2.2} e^x dx$ by using 4-segment Simpson's 1/3 rule most nearly is

(A) 7.8036

(B) 7.8062

(C) 7.8423

(D) 7.9655

Solution

The correct answer is (B).

The multiple segment equation for Simpson's 1/3 rule is

$$\int_a^b f(x)dx \approx \frac{b-a}{3n}\left[f(x_0) + 4\sum_{\substack{i=1 \\ i=\text{odd}}}^{n-1} f(x_i) + 2\sum_{\substack{i=2 \\ i=\text{even}}}^{n-2} f(x_i) + f(x_n) \right]$$

Using 4 segments gives

$$a = 0.2$$

$$b = 2.2$$

$$n = 4$$

$$h = \frac{b-a}{n}$$

$$= \frac{2.2-0.2}{4}$$

$$= 0.5$$

$$\int_{0.2}^{2.2} e^x dx \approx \frac{2.2-0.2}{3\times 4}\left[f(x_0) + 4\sum_{\substack{i=1 \\ i=\text{odd}}}^{4-1} f(x_i) + 2\sum_{\substack{i=2 \\ i=\text{even}}}^{4-2} f(x_i) + f(x_4) \right]$$

$$= \frac{2.2-0.2}{3\times 4}\left[f(x_0) + 4\sum_{\substack{i=1 \\ i=\text{odd}}}^{3} f(x_i) + 2\sum_{\substack{i=2 \\ i=\text{even}}}^{2} f(x_i) + f(x_4) \right]$$

$$= \frac{2.2-0.2}{3\times 4}\left[f(x_0) + 4\left(f(x_1) + f(x_3)\right) + 2\left(f(x_2)\right) + f(x_4) \right]$$

So

$$f(x) = e^x$$
$$f(x_0) = f(0.2) = e^{0.2} = 1.2214$$

$$f(x_1) = f(0.2 + 0.5) = f(0.7)$$
$$f(0.7) = e^{0.7} = 2.0138$$

$$f(x_2) = f(0.7 + 0.5) = f(1.2)$$
$$f(1.2) = e^{1.2} = 3.3201$$

$$f(x_3) = f(1.2 + 0.5) = f(1.7)$$
$$f(1.7) = e^{1.7} = 5.4739$$

$$f(x_4) = f(2.2) = e^{2.2} = 9.0250$$

$$\begin{aligned}
\int_{0.2}^{2.2} e^x dx &\approx \frac{2.2 - 0.2}{3 \times 4}\left[f(x_0) + 4\left(f(x_1) + f(x_3)\right) + 2\left(f(x_2)\right) + f(x_4)\right] \\
&= \frac{2.2 - 0.2}{3 \times 4}\left[1.2214 + 4(2.0138 + 5.4739) + 2(3.3201) + 9.0250\right] \\
&= 0.16667\left[1.2214 + 29.951 + 6.6402 + 9.0250\right] \\
&= 7.8062
\end{aligned}$$

4. The velocity of a body is given by

$$v(t) = 2t, \qquad 1 \le t \le 5$$
$$= 5t^2 + 3, \quad 5 < t \le 14$$

where t is given in seconds, and v is given in m/s. Using two-segment Simpson's 1/3 rule, the distance in meters covered by the body from $t = 2$ to $t = 9$ seconds most nearly is

(A) 949.33
(B) 1039.7
(C) 1200.5
(D) 1442.0

Solution

The correct answer is (C).

The multiple segment equation for Simpson's 1/3 rule is

$$\int_a^b f(x)dx \approx \frac{b-a}{3n}\left[f(x_0) + 4\sum_{\substack{i=1 \\ i=\text{odd}}}^{n-1} f(x_i) + 2\sum_{\substack{i=2 \\ i=\text{even}}}^{n-2} f(x_i) + f(x_n)\right]$$

$$a = 2$$
$$b = 9$$
$$n = 2$$
$$h = \frac{b-a}{n}$$
$$= \frac{9-2}{2}$$
$$= 3.5$$

$$\int_2^9 v(t)dt \approx \frac{9-2}{3\times 2}\left[v(t_0) + 4\sum_{\substack{i=1 \\ i=\text{odd}}}^{2-1} v(t_i) + 2\sum_{\substack{i=2 \\ i=\text{even}}}^{2-2} v(t_i) + v(t_2)\right]$$

$$= \frac{9-2}{3\times 2}\left[v(t_0) + 4\sum_{\substack{i=1 \\ i=\text{odd}}}^{1} v(t_i) + 2\sum_{\substack{i=2 \\ i=\text{even}}}^{0} v(t_i) + v(t_2)\right]$$

$$= \frac{9-2}{3\times 2}\left[v(t_0) + 4(v(t_1)) + v(t_2)\right]$$

So

$$v(t) = 2t, \qquad 1 \le t \le 5$$
$$= 5t^2 + 3, \quad 5 < t \le 14$$

$$v(t_0) = v(2) = 2 \times 2 = 4 \text{ m/s}$$

$$v(t_1) = v(2 + 3.5) = v(5.5)$$
$$v(5.5) = 5 \times 5.5^2 + 3 = 154.25 \text{ m/s}$$

$$v(t_2) = v(9) = 5 \times 9^2 + 3 = 408 \text{ m/s}$$

$$\int_2^9 v(t)dt \approx \frac{9-2}{3 \times 2}\left[v(t_0) + 4 \times v(t_1) + v(t_2)\right]$$
$$= \frac{9-2}{3 \times 2}\left[v(2) + 4 \times v(5.5) + v(9)\right]$$
$$= 1.1667\left[4 + 4 \times 154.25 + 408\right]$$
$$= 1200.5 \text{ m}$$

5. The value of $\int_{3}^{19} f(x)dx$ by using 2-segment Simpson's 1/3 rule is estimated as 702.039. The estimate of the same integral using 4-segment Simpson's 1/3 rule most nearly is

(A) $702.039+\frac{8}{3}\left[2f(7)-f(11)+2f(15)\right]$

(B) $\frac{702.039}{2}+\frac{8}{3}\left[2f(7)-f(11)+2f(15)\right]$

(C) $702.039+\frac{8}{3}\left[2f(7)+2f(15)\right]$

(D) $\frac{702.039}{2}+\frac{8}{3}\left[2f(7)2f(15)\right]$

Solution

The correct answer is (B).

Using 2-segment Simpson's 1/3 rule gives

$$\int_{3}^{19} f(x)dx \approx \frac{19-3}{3\times 2}\left[f(x_0)+4\sum_{\substack{i=1\\ i=\text{odd}}}^{2-1} f(x_i)+2\sum_{\substack{i=2\\ i=\text{even}}}^{2-2} f(x_i)+f(x_2)\right]$$

$$=\frac{19-3}{3\times 2}\left[f(x_0)+4\sum_{\substack{i=1\\ i=\text{odd}}}^{1} f(x_i)+f(x_2)\right]$$

$$=\frac{19-3}{3\times 2}\left[f(x_0)+4f(x_1)+f(x_2)\right]$$

$$702.039 \approx \frac{19-3}{3\times 2}\left[f(3)+4f(11)+f(19)\right]$$

Using 4-segment Simpson's 1/3 rule gives

$$\int_{3}^{19} f(x)dx \approx \frac{19-3}{3\times 4}\left[f(x_0)+4\sum_{\substack{i=1\\ i=\text{odd}}}^{4-1} f(x_i)+2\sum_{\substack{i=2\\ i=\text{even}}}^{4-2} f(x_i)+f(x_4)\right]$$

$$=\frac{19-3}{3\times 4}\left[f(x_0)+4\sum_{\substack{i=1\\ i=\text{odd}}}^{3} f(x_i)+2\sum_{\substack{i=2\\ i=\text{even}}}^{2} f(x_i)+f(x_4)\right]$$

$$=\frac{19-3}{3\times 4}\left[f(x_0)+4\left(f(x_1)+f(x_3)\right)+2\left(f(x_2)\right)+f(x_4)\right]$$

$$=\frac{19-3}{3\times 4}\left[f(3)+4\left(f(7)+f(15)\right)+2\left(f(11)\right)+f(19)\right]$$

$$= \frac{19-3}{3\times 4}\left[f(3)+4f(7)+4f(11)+4f(15)-2f(11)+f(19)\right]$$

$$= \frac{19-3}{3\times 4}\left[f(3)+4f(11)+f(19)\right]+\frac{19-3}{3\times 4}\left[4f(7)+4f(15)-2f(11)\right]$$

$$= \frac{702.039}{2}+\frac{19-3}{3\times 4}\left[4f(7)+4f(15)-2f(11)\right]$$

$$= \frac{702.039}{2}+\frac{2(19-3)}{3\times 4}\left[2f(7)+2f(15)-f(11)\right]$$

$$= \frac{702.039}{2}+\frac{8}{3}\left[2f(7)+2f(15)-f(11)\right]$$

6. The following data of the velocity of a body is given as a function of time.

Time (s)	4	7	10	15
Velocity (m/s)	22	24	37	46

The best estimate of the distance in meters covered by the body from $t = 4$ to $t = 15$ using combined Simpson's 1/3 rule and the trapezoidal rule would be

(A) 354.70

(B) 362.50

(C) 368.00

(D) 378.80

Solution

The correct answer is (B).

$$t_0 = 4,\ t_1 = 7,\ t_2 = 10,\ t_3 = 15$$

We can use Simpson's 1/3 rule from $t = 4$ to $t = 10$ as we have three equidistant points, $t = 4,7,10$.

$$\int_a^b v(t)dt \approx \frac{b-a}{3n}\left[v(t_0) + 4\sum_{\substack{i=1 \\ i=\text{odd}}}^{n-1} v(t_i) + 2\sum_{\substack{i=2 \\ i=\text{even}}}^{n-2} v(t_i) + v(t_n)\right]$$

where

$$a = t_0$$

$$b = t_2$$

$$h = t_2 - t_1$$

$$= t_1 - t_0$$

$$= 3$$

$$h = \frac{b-a}{n}$$

$$3 = \frac{10-4}{n}$$

$$n = 2$$

Thus, using 2-segment Simpson's 1/3 rule

$$\int_{t_0}^{t_2} v(t)dt \approx \frac{t_2 - t_0}{3 \times 2}\left[v(t_0) + 4v(t_1) + v(t_2)\right]$$

Using the trapezoidal rule with unequal segments from $t = 10$ to $t = 15$

$$\int_{t_2}^{t_3} v(t)dt \approx (t_3 - t_2)\left[\frac{v(t_2) + v(t_3)}{2}\right]$$

Thus,

$$\int_{t_0}^{t_3} v(t)dt \approx \frac{t_2 - t_0}{3\times 2}\left[v(t_0) + 4v(t_1) + v(t_2)\right] + (t_3 - t_2)\left[\frac{v(t_2) + v(t_3)}{2}\right]$$

$$\int_{4}^{14} v(t)dt \approx \frac{10 - 4}{3\times 2}\left[v(4) + 4v(7) + v(10)\right] + (15 - 10)\left[\frac{v(10) + v(15)}{2}\right]$$

$$= \frac{10 - 4}{3\times 2}\left[22 + 4\times 24 + 37\right] + (15 - 10)\left[\frac{37 + 46}{2}\right]$$

$$= (6)\left[25.833\right] + (5)\left[41.5\right]$$

$$= 362.5 \text{ m}$$