Multiple Choice Test

Chapter 07.06
Discrete Data Integration

1. Given below is the discrete data of y vs x.

x	0	10	15	20
y	100.00	227.04	362.78	517.35

Amongst the following methods, which one would be the best scientific method to use to find the integral $\int_0^{20} y dx$?

(A) Average method
(B) Trapezoidal rule with unequal segments
(C) $y(10) \times (20 - 0)$
(D) $y(20) \times 20$

Solution
The correct answer is (B).

Using the average method is no longer the best scientific method based on your current knowledge. Trapezoidal rule with unequal segments is a better choice. Choice C does not use the value of y at three other points, and without reason uses the $y(10)$ as the mean value of y between 0 and 20. Choice D makes the incorrect assumption that $y(20)$ is the mean value of y between 0 and 20

2. Given $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ data pairs where $x_1 < x_2 < x_3$, then $\int_{x_1}^{x_3} y(x)dx$ can be best approximated as

(A) $\frac{y_1 + y_2 + y_3}{3}(x_3 - x_1)$

(B) $\frac{y_1 + y_2}{2}(x_3 - x_1)$

(C) $(x_2 - x_1)(\frac{y_1 + y_2}{2}) + (x_3 - x_2)(\frac{y_2 + y_3}{2})$

(D) $y_2(x_3 - x_1)$

Solution

The correct solution is (C).

The best answer amongst the above given choices is using the trapezoidal rule with unequal segments. The general formula for this rule for *n* data pairs $(x_1, y_1), (x_2, y_2), ..., (x_n, y_n)$ is given by

$$\int_{x_1}^{x_n} y(x)dx = \sum_{i=1}^{n-1}(x_{i+1} - x_i)\left(\frac{y(x_i) + y(x_{i+1})}{2}\right)$$

For *n*=3,

$$\int_{x_1}^{x_3} y(x)dx = \sum_{i=1}^{2}(x_{i+1} - x_i)\left(\frac{y(x_i) + y(x_{i+1})}{2}\right)$$

$$= (x_2 - x_1)\left(\frac{y(x_1) + y(x_2)}{2}\right) + (x_3 - x_2)\left(\frac{y(x_2) + y(x_3)}{2}\right)$$

$$= (x_2 - x_1)\left(\frac{y_1 + y_2}{2}\right) + (x_3 - x_2)\left(\frac{y_2 + y_3}{2}\right)$$

Choice A simply uses the average of the given function values to calculate the integral, irrespective of the points where these values are given at. Choice B does not even use all the points to calculate the average function value. Choice D assumes that the middle value is a good approximation of the average value of the function between x_1 and x_3.

3. The following data of the velocity of a body as a function of time is given.

Time, t (s)	0	15	18	22	24
Velocity, v (m/s)	22	24	31.4	25	123

Using the trapezoidal rule with unequal segments, the distance covered in meters by the body from t=15 to 22 seconds is

(A) 171.5
(B) 187.6
(C) 195.9
(D) 204.9

Solution

The correct solution is (C).

The distance covered by the body between 15 and 22 seconds is given by

$$\begin{aligned} s(22)-s(15) &= \int_{22}^{15} v(t)dt \\ &= \int_{15}^{18} v(t)dt + \int_{18}^{22} v(t)dt \\ &\approx (18-15)\left(\frac{v(18)+v(15)}{2}\right)+(22-18)\left(\frac{v(22)+v(18)}{2}\right) \\ &= (3)\left(\frac{31.4+24}{2}\right)+(4)\left(\frac{25+31.4}{2}\right) \\ &= 195.9\,m \end{aligned}$$

4. The following data of the velocity of a body as a function of time is given.

Time, t (s)	0	15	18	22	24
Velocity, v (m/s)	22	24	37	25	123

Using the trapezoidal rule with unequal segments, the distance covered in meters by the body from t=9.3 to 18 seconds is

(A) 226.1
(B) 240.7
(C) 262.0
(D) 436.5

Solution

The correct solution is (A).

Using the trapezoidal rule with unequal segments, the distance covered by the body from 9.3 to 18 seconds is

$$s(18) - s(9.3) = \int_{9.3}^{18} v(t)dt$$

$$= \int_{9.3}^{15} v(t)dt + \int_{15}^{18} v(t)dt$$

$$\cong (15 - 9.3)\left(\frac{v(15) + v(9.3)}{2}\right) + (18 - 15)\left(\frac{v(18) + v(15)}{2}\right)$$

To find the unknown $v(9.3)$, we use the linear interpolation formula

$$v(t) = \frac{v(t_1) - v(t_0)}{t_1 - t_0}(t - t_0) + v(t_0),\ t_0 \le t \le t_i$$

Using $t = 9.3, t_0 = 0, t_1 = 15$ in the above formula, we get,

$$v(9.3) = \frac{v(15) - v(0)}{15 - 0}(9.3 - 0) + v(0)$$

$$= \frac{24 - 22}{15 - 0}(9.3) + 22$$

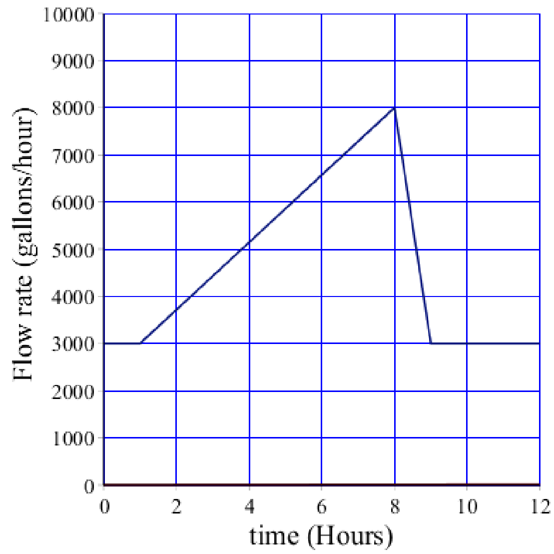
$$= 23.24\, m/s$$

Hence

$$s(18) - s(9.3) \cong (15 - 9.3)\frac{24 + 23.24}{2} + (18 - 15)\frac{37 + 24}{2}$$

$$= 226.1m$$

5. The flow rate of oil is given in gallons per hour through a pipeline over a 12-hour period as shown.



The best approximation of the number of gallons of oil that has flowed through the 12-hour period is

(A) 48000
(B) 56000
(C) 66000
(D) 86148

Solution

The correct solution is (B).

To find the volume of oil Q produced in the 12 hour period, we need to integrate the flow rate from 0 to 12 hours,

$$Q = \int_0^{12} \dot{Q}(t)dt$$

where

$\dot{Q}(t) =$ Flow rate (gallons/hr)

$$Q = \int_0^{12} \dot{Q}(t)dt$$

$$= \int_0^{1} \dot{Q}(t) + \int_1^{8} \dot{Q}(t)dt + \int_8^{9} \dot{Q}(t)dt + \int_9^{12} \dot{Q}(t)dt$$

$$= (1-0)\left(\frac{Q(0)+Q(1)}{2}\right) + (8-1)\left(\frac{Q(1)+Q(8)}{2}\right)$$

$$+ (9-8)\left(\frac{Q(8)+Q(9)}{2}\right) + (12-9)\left(\frac{Q(9)+Q(12)}{2}\right)$$

$$= (1-0)\left(\frac{3000+3000}{2}\right) + (8-1)\left(\frac{3000+8000}{2}\right)$$

$$+ (9-8)\left(\frac{8000+3000}{2}\right) + (12-9)\left(\frac{3000+3000}{2}\right)$$

$$= 56000 \;\; gallons$$

6. Water is flowing through a circular pipe of 0.5 ft radius, and flow velocity (ft/s) measurements are made from the center to the wall of the pipe as follows.

Radial Location, *r (ft)*	0	0.17	0.33	0.50
Velocity, *v (ft/s)*	10	8.8	5.6	0

Note that the volumetric flow rate, $\dot{Q}$ is given by

$$\dot{Q} = \int_0^a 2\pi r v(r) dr$$

where $v(r)$ is the flow velocity of the fluid as a function of the radial location, r, and a is the radius of the pipe. The volumetric flow rate in ft^3/s in the pipe by using trapezoidal rule with unequal segments is

(A) 3.226
(B) 3.469
(C) 3.901
(D) 20.27

Solution

The correct solution is (B)

The water flow rate is given by

$$Q = \int_0^a 2\pi r v(r) dr$$

$$= 2\pi \int_0^{0.5} r v(r) dr$$

$$= 2\pi \left(\int_0^{0.17} r v(r) dr + \int_{0.17}^{0.33} r v(r) dr + \int_{0.33}^{0.5} r v(r) dr \right)$$

$$\approx 2\pi \left[\begin{array}{l} (0.17-0)\left(\dfrac{0.17 \times v(0.17) + 0 \times v(0)}{2} \right) \\ +(0.33-0.17)\left(\dfrac{0.33 \times v(0.33) + 0.17 \times v(0.17)}{2} \right) \\ +(0.5-0.33)\left(\dfrac{0.5 \times v(0.5) + 0.33 \times v(0.33)}{2} \right) \end{array} \right]$$

$$\approx 2\pi \left[\begin{array}{l} (0.17-0)\left(\dfrac{0.17 \times 8.8 + 0 \times 10}{2} \right) \\ +(0.33-0.17)\left(\dfrac{0.33 \times 5.6 + 0.17 \times 8.8}{2} \right) \\ +(0.5-0.33)\left(\dfrac{0.5 \times 0 + 0.33 \times 5.6}{2} \right) \end{array} \right]$$

$$= 2\pi\left[(0.17)(0.7480) + (0.16)(1.672) + (0.17)(0.9240)\right]$$

$$= 3.469\frac{m^3}{s}$$