Holistic Numerical Methods Institute
committed to bringing numerical methods to undergraduates

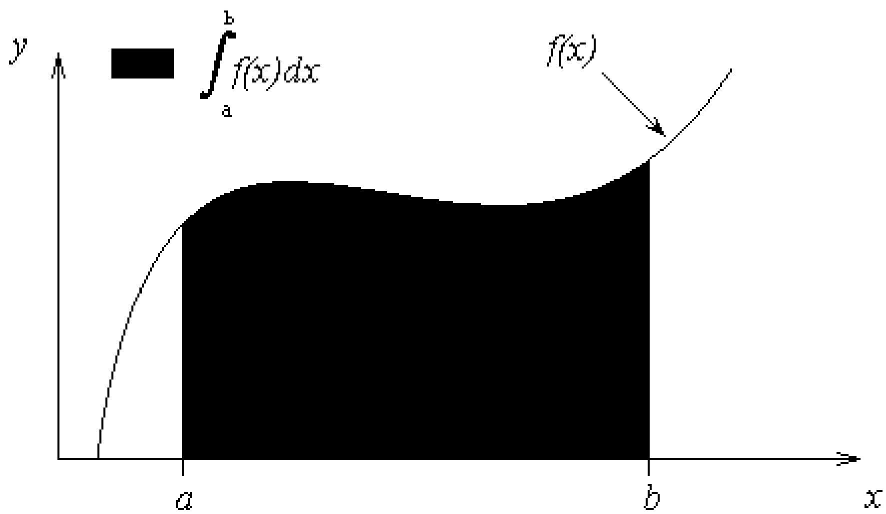
Multiple-Choice Test
Background
Integration
COMPLETE SOLUTION SET

1. Physically, integrating $\int_a^b f(x)dx$ means finding the

   (A) area under the curve from *a* to *b*
   (B) area to the left of point *a*
   (C) area to the right of point *b*
   (D) area above the curve from *a* to *b*

Solution
The correct answer is (A).

Integrating $\int_a^b f(x)dx$ means finding the area under the curve of the function $f(x)$ from *a* to *b*.



2. The mean value of a function $f(x)$ from a to b is given by

(A) $\dfrac{f(a)+f(b)}{2}$

(B) $\dfrac{f(a)+2f\left(\dfrac{a+b}{2}\right)+f(b)}{4}$

(C) $\int_a^b f(x)dx$

(D) $\dfrac{\int_a^b f(x)dx}{b-a}$

Solution

The correct answer is (D).

The mean value of a function $f(x)$ from a to b is given by

$$\bar{f} = \frac{\text{Area under the curve from } a \text{ to } b}{\text{Width of the interval from } a \text{ to } b}$$

$$= \frac{\int_a^b f(x)dx}{b-a}$$

3. The exact value of $\int_{0.2}^{2.2} xe^x dx$ is most nearly

(A) 7.8036
(B) 11.807
(C) 14.034
(D) 19.611

Solution
The correct answer is (B).

To solve this integral we must integrate by parts.

$$\int u\,dv = uv - \int v\,du$$

where

$$u = x$$

$$du = dx$$

and

$$dv = e^x dx$$

$$v = e^x$$

$$\int_{0.2}^{2.2} xe^x dx = \int_{0.2}^{2.2} xd\left(e^x\right)$$

$$= \left[xe^x\right]_{0.2}^{2.2} - \int_{0.2}^{2.2} e^x dx$$

$$= \left[xe^x - e^x\right]_{0.2}^{2.2}$$

$$= \left(2.2e^{2.2} - e^{2.2}\right) - \left(0.2e^{0.2} - e^{0.2}\right)$$

$$= 10.83 - (-0.9771)$$

$$= 11.807$$

4. $\int_{0.2}^{2} f(x)dx$ for

$$f(x) = x, \quad 0 \le x \le 1.2$$
$$= x^2, \quad 1.2 < x \le 2.4$$

is most nearly

(A) 1.9800
(B) 2.6640
(C) 2.7907
(D) 4.7520

Solution

The correct answer is (C).

$$\int_{0.2}^{2} f(x)dx = \int_{0.2}^{1.2} x\ dx + \int_{1.2}^{2} x^2 dx$$

$$= \left[\frac{1}{2}x^2\right]_{0.2}^{1.2} + \left[\frac{1}{3}x^3\right]_{1.2}^{2}$$

$$= \left(\frac{1.2^2}{2} - \frac{0.2^2}{2}\right) + \left(\frac{2^3}{3} - \frac{1.2^3}{3}\right)$$

$$= 0.7 + 2.0907$$

$$= 2.7907$$

5. The area of a circle of radius a can be found by the following integral

(A) $\int_0^a (a^2 - x^2)dx$

(B) $\int_0^{2\pi} \sqrt{a^2 - x^2}dx$

(C) $4\int_0^a \sqrt{a^2 - x^2}dx$

(D) $\int_0^a \sqrt{a^2 - x^2}dx$

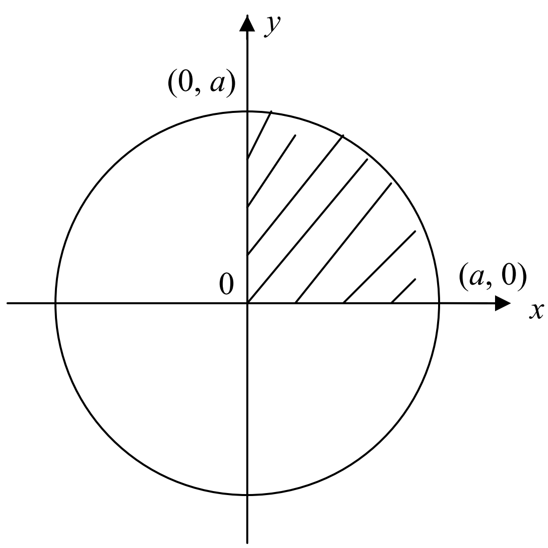
Solution

The correct answer is (C).

The equation for a circle of radius a is

$$y^2 + x^2 = a^2$$

$$y = \sqrt{a^2 - x^2}$$



To find the area of the shaded quarter circle shown, we can find the area under the curve of the integral of $y = \sqrt{a^2 - x^2}$ from 0 to a.

$$A = \int_0^a \sqrt{a^2 - x^2}dx$$

Thus, the area of the full circle is four times the quarter circle

$$A = 4\int_0^a \sqrt{a^2 - x^2}dx$$

6. Velocity distribution of a fluid flow through a pipe varies along the radius and is given by $v(r)$. The flow rate through the pipe of radius a is given by

(A) $\pi v(a)a^2$

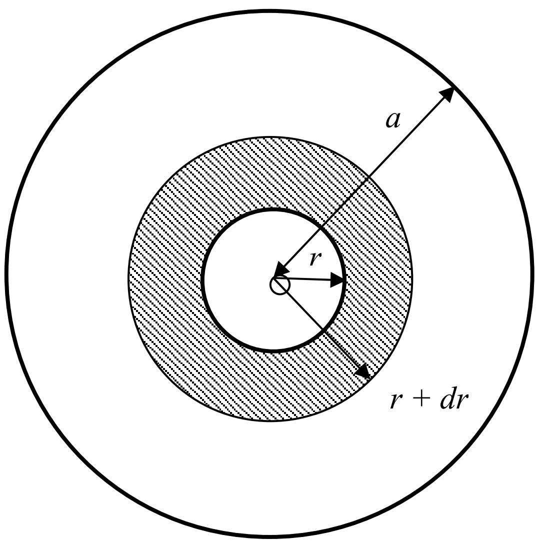
(B) $\pi \dfrac{v(0)+v(a)}{2}a^2$

(C) $\int_0^a v(r)dr$

(D) $2\pi\int_0^a v(r)rdr$

Solution

The correct answer is (D).



The differential area is the area bound by the concentric circles and is given by

$$dA = \pi(r+dr)^2 - \pi r^2$$
$$= \pi r^2 + \pi(dr)^2 + 2\pi r\, dr - \pi r^2$$
$$= 2\pi r\, dr + \pi(dr)^2$$

Since

$$(dr)^2 << dr$$

we have

$$dA \approx 2\pi r\ dr$$

The flow rate $d\dot{Q}$ through the differential element of the pipe is

$$
\begin{aligned}
d\dot{Q} &= (velocity)\times(differential\ area) \\
&= (v(r))\times(2\pi r\,dr)
\end{aligned}
$$

$$
\begin{aligned}
\dot{Q} &= \int_0^a v(r)2\pi r\ dr \\
&= 2\pi\int_0^a v(r)r\ dr
\end{aligned}
$$