Multiple-Choice Test

Chapter 06.04
Non-Linear Regression

1. When using the transformed data model to find the constants of the regression model \( y = ae^{bx} \) to best fit \((x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\), the sum of the square of the residuals that is minimized is

(A) \( \sum_{i=1}^{n} \left( y_i - ae^{bx_i} \right)^2 \)

(B) \( \sum_{i=1}^{n} \left( \ln(y_i) - \ln(a) - bx_i \right)^2 \)

(C) \( \sum_{i=1}^{n} \left( y_i - \ln(a) - bx_i \right)^2 \)

(D) \( \sum_{i=1}^{n} \left( \ln(y_i) - \ln(a) - b \ln(x_i) \right)^2 \)

2. It is suspected from theoretical considerations that the rate of water flow from a firehouse is proportional to some power of the nozzle pressure. Assume pressure data is more accurate. You are transforming the data.

<table>
<thead>
<tr>
<th>Flow rate, ( F ) (gallons/min)</th>
<th>96</th>
<th>129</th>
<th>135</th>
<th>145</th>
<th>168</th>
<th>235</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure, ( p ) (psi)</td>
<td>11</td>
<td>17</td>
<td>20</td>
<td>25</td>
<td>40</td>
<td>55</td>
</tr>
</tbody>
</table>

The exponent of the nozzle pressure in the regression model \( F = ap^b \) most nearly is

(A) 0.49721
(B) 0.55625
(C) 0.57821
(D) 0.67876

3. The transformed data model for the stress-strain curve \( \sigma = k_1 \varepsilon e^{-k_2 \varepsilon} \) for concrete in compression, where \( \sigma \) is the stress and \( \varepsilon \) is the strain, is

(A) \( \ln(\sigma) = \ln(k_1) + \ln(\varepsilon) - k_2 \varepsilon \)

(B) \( \ln(\frac{\sigma}{\varepsilon}) = \ln(k_1) - k_2 \varepsilon \)

(C) \( \ln(\frac{\sigma}{\varepsilon}) = \ln(k_1) + k_2 \varepsilon \)

(D) \( \ln(\sigma) = \ln(k_1 \varepsilon) - k_2 \varepsilon \)
4. In nonlinear regression, finding the constants of the model requires solving simultaneous nonlinear equations. However in the exponential model \( y = ae^{bx} \) that is best fit to \( (x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n) \), the value of \( b \) can be found as a solution of a single nonlinear equation. That nonlinear equation is given by

\[
\sum_{i=1}^{n} y_i x_i e^{bx_i} - \sum_{i=1}^{n} y_i e^{bx_i} \sum_{i=1}^{n} x_i = 0
\]

(A)

\[
\sum_{i=1}^{n} y_i x_i e^{bx_i} = \sum_{i=1}^{n} y_i e^{bx_i} \sum_{i=1}^{n} x_i e^{2bx_i} = 0
\]

(B)

\[
\sum_{i=1}^{n} y_i x_i e^{bx_i} = \sum_{i=1}^{n} y_i e^{bx_i} \sum_{i=1}^{n} e^{bx_i} = 0
\]

(C)

\[
\sum_{i=1}^{n} y_i x_i e^{bx_i} = \sum_{i=1}^{n} y_i e^{bx_i} \sum_{i=1}^{n} x_i e^{2bx_i} = 0
\]

(D)

\[
\sum_{i=1}^{n} y_i e^{bx_i} - \sum_{i=1}^{n} x_i e^{2bx_i} = 0
\]

5. There is a functional relationship between the mass density \( \rho \) of air and the altitude \( h \) above the sea level.

<table>
<thead>
<tr>
<th>Altitude above sea level, ( h ) (km)</th>
<th>0.32</th>
<th>0.64</th>
<th>1.28</th>
<th>1.60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass Density, ( \rho ) (kg/m(^3))</td>
<td>1.15</td>
<td>1.10</td>
<td>1.05</td>
<td>0.95</td>
</tr>
</tbody>
</table>

In the regression model \( \rho = k_1 e^{-k_2 h} \), the constant \( k_2 \) is found as \( k_2 = 0.1315 \). Assuming the mass density of air at the top of the atmosphere is \( 1/1000 \) of the mass density of air at sea level. The altitude in kilometers of the top of the atmosphere most nearly is

(A) 46.2
(B) 46.6
(C) 49.7
(D) 52.5
6. A steel cylinder at 80°F of length 12" is placed in a commercially available liquid nitrogen bath (−315°F). If the thermal expansion coefficient of steel behaves as a second order polynomial function of temperature and the polynomial is found by regressing the data below,

<table>
<thead>
<tr>
<th>Temperature, $T$ (°F)</th>
<th>Thermal expansion Coefficient, $\alpha$ (µ in/in/°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>−320</td>
<td>2.76</td>
</tr>
<tr>
<td>−240</td>
<td>3.83</td>
</tr>
<tr>
<td>−160</td>
<td>4.72</td>
</tr>
<tr>
<td>−80</td>
<td>5.43</td>
</tr>
<tr>
<td>0</td>
<td>6.00</td>
</tr>
<tr>
<td>80</td>
<td>6.47</td>
</tr>
</tbody>
</table>

the reduction in the length of the cylinder in inches most nearly is

(A) 0.0219
(B) 0.0231
(C) 0.0235
(D) 0.0307

For a complete solution, refer to the links at the end of the book.