Holistic Numerical Methods Institute

committed to bringing numerical methods to undergraduates

Multiple-Choice Test
Background
Regression

COMPLETE SOLUTION SET

1). The average of 7 numbers is given 12.6. If 6 of the numbers are $5, 7, 9, 12, 17$ and 10, the remaining number is

(A) -47.9

(B) -47.4

(C) 15.6

(D) 28.2

Solution

The correct answer is (D)

If x is the remaining number, then

$$\frac{5+7+9+12+17+10+x}{7} = 12.6$$

$$\begin{aligned} x &= (12.6 \times 7) - (5+7+9+12+17+10) \\ &= 88.2 - (60) \\ &= 28.2 \end{aligned}$$

2). The average and standard deviation of 7 numbers is given a 8.142 and 5.005, respectively. If 5 numbers are $5, 7, 9, 12$ and 17, the other two numbers are

(A) $-0.1738, 7.175$

(B) $3.396, 12.890$

(C) $3.500, 3.500$

(D) $4.488, 2.512$

Solution

The correct answer is (D)

Let *x* and *y* be the two missing numbers.

From the average of the numbers being 8.142, we have

$$\frac{5+7+9+12+17+x+y}{7} = 8.142$$
$$x + y = (8.142 \times 7) - (5+7+9+12+17)$$
$$x + y = 7 \qquad (1)$$

From the standard deviation being 5.005, we have

$$\sqrt{\frac{\begin{array}{c}(5-8.142)^2 + (7-8.142)^2 + (9-8.142)^2 + (12-8.142)^2 \\ +(17-8.142)^2 + (x-8.142)^2 + (y-8.142)^2\end{array}}{7-1}} = 5.005$$

$$(x-8.142)^2 + (y-8.142)^2 =$$
$$(5.005^2 \times 6) - ((5-8.142)^2 + (7-8.142)^2 + (9-8.142)^2 + (12-8.142)^2 + (17-8.142)^2)$$

$$x^2 - 16.284x + 66.292 + y^2 - 16.284y + 66.292 = 45.039$$
$$x^2 + y^2 - 16.284(x+y) = -87.544 \qquad (2)$$

From Equation (1)

$$x = 7 - y$$

then

$$(7-y)^2 + y^2 - 16.284(7-y) - 16.2840y = -87.544$$
$$y^2 - 14y + 49 + y^2 + 16.284y - 16.284y - 113.988 = -87.544$$
$$2y^2 - 14y - 64.988 = -87.544$$
$$2y^2 - 14y + 22.556 = 0$$

Using the quadratic equation solution

$$y = \frac{-(-14) \pm \sqrt{(-14)^2 - 4 \times 2 \times 22.556}}{2 \times 2}$$
$$= \frac{14 \pm \sqrt{15.552}}{4}$$

gives

$$y = 4.488$$

or

$$y = 2.512$$

Thus,

$$x = 2.512$$

or

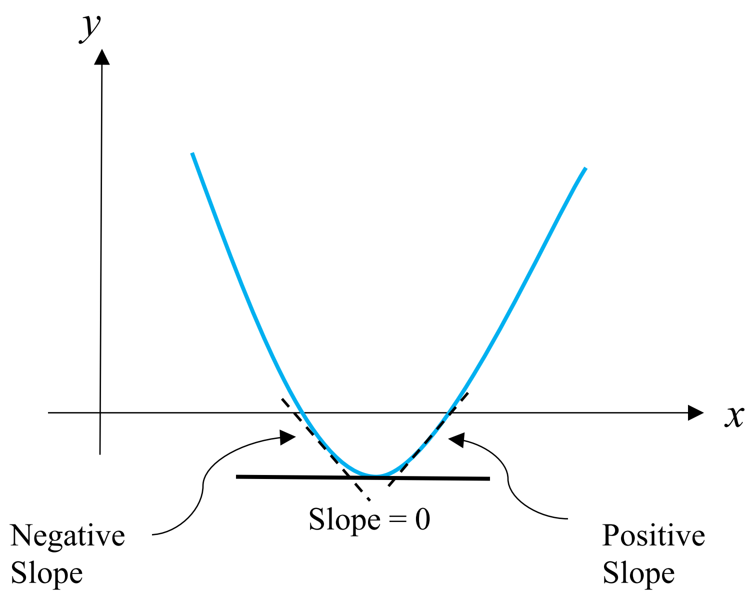
$$x = 4.488$$

Hence $(x, y) = (4.488, 2.512)$ or $(2.512, 4.488)$, which are the same pair of numbers.

3). A local minimum of a continuous function in the interval $(-\infty, \infty)$ exists at $x = a$ if
(A) $f'(a) = 0, f''(a) = 0$
(B) $f'(a) = 0, f''(a) < 0$
(C) $f'(a) = 0, f''(a) > 0$
(D) $f'(a) = 0, f''(a)$ does not exist

Solution
The correct answer is (C)

A continuous function whose slope $f'(x)$ is zero at $x = a$ will correspond to a local minimum if the slope at $x = a-$ is negative and the slope at $x = a+$ is positive. This would imply $f'' > 0$.



4). The absolute minimum of a function $f(x) = x^2 + 2x - 15$ in the interval $(-\infty, \infty)$ exists at $x =$________ and is ________.
(A) $x = -1, f(-1) = -16$
(B) $x = -1, f(-1) = 0$
(C) $x = 3, f(3) = 0$
(D) $x = 5, f(5) = 0$

Solution
The correct answer is (A)

Given

$$f(x) = x^2 + 2x - 15$$

then

$$f'(x) = 2x + 2$$

To find the critical points, put

$$f'(x) = 0$$

gives

$$2x + 2 = 0$$
$$x = -1$$

Since the first derivative, $f'(x)$ is defined in the domain $(-\infty, \infty)$, and $f'(x) = 0$ at $x = -1$, it is the only critical point.

$$f''(x) = 2$$
$$f''(-1) = 2$$

And since $f''(-1) > 0$, the critical point $x = -1$ corresponds to a local minimum.

Since $f'(x) = 0$ at only one point $x = -1$ and the function $f(x)$ is continuous in $(-\infty, \infty)$, it also corresponds to the absolute minimum. So, the absolute minimum exists at $x = -1$ and it is

$$f(-1) = (-1)^2 + 2(-1) - 15$$
$$= -16$$

5). The first order partial derivative with respect to x of $u(x, y) = x^2y^3 + 6x^3e^{2y}$

(A) $y^3 + 6e^{2y}$
(B) $3x^2y^2 + 18x^3e^{2y}$
(C) $2xy^3 + 18x^2e^{2y}$
(D) $2xy^3 + 24x^2e^{2y}$

Solution

The correct answer is (C)

$$
\begin{aligned}
u(x, y) &= x^2y^3 + 6x^3e^{2y} \\
\frac{\partial y}{dx} &= \frac{\partial}{dx}(x^2y^3 + 6x^3e^{2y}) \\
&= y^3\frac{\partial}{dx}(x^2) + e^{2y}\frac{\partial}{dx}(6x^3) \\
&= y^3(2x) + e^{2y}(18x^2) \\
&= 2xy^3 + 18x^2e^{2y}
\end{aligned}
$$

(6). The critical point(s) (x, y) of the function $f(x, y) = y^3 + 4xy - 16y - 4x^2$ is (are)
(A) $(-4/3,1), (-8/3,2)$
(B) $(4/3,8/3), (-1,-2)$
(C) $(-4/3,-8/3), (1,2)$
(D) $(0,0)$

Solution
The correct answer is (C)

$$f(x, y) = y^3 + 4xy - 16y - 4x^2$$

The critical points are where

$$\frac{\partial f}{dx} = 0 \text{ and } \frac{\partial f}{dy} = 0$$

or where

$$\frac{\partial f}{dx} \text{ or } \frac{\partial f}{dy} \text{ do not exist}$$

$$\frac{\partial f}{dx} = 4y - 8x$$
$$\frac{\partial f}{dy} = 3y^2 + 4x - 16$$

Since $\frac{\partial f}{dx}$ and $\frac{\partial f}{dy}$ are defined everywhere in $(-\infty, \infty)$, we only need to seek points where

$$\frac{\partial f}{dx} = 0 \text{ and } \frac{\partial f}{dy} = 0$$

$$\frac{\partial f}{dx} = 0$$
$$4y - 8x = 0$$
$$y = 2x$$

$$\frac{\partial f}{dy} = 0$$
$$3y^2 + 4x - 16 = 0$$
$$3(2x)^2 + 4x - 16 = 0$$
$$12x^2 + 4x - 16 = 0$$
$$x = \frac{-4 \pm \sqrt{16 - 4(12)(-16)}}{2(12)}$$
$$= -\frac{4}{3}, 1$$

From $y = 2x$, the corresponding values of y are

$$y = -\frac{8}{3}, 2$$

So $\left(-\frac{4}{3}, -\frac{8}{3}\right)$ and $(1,2)$ are the two critical points.