Multiple-Choice Test

Chapter 04.08
Gauss-Seidel Method

1. A square matrix $[A]_{n\times n}$ is diagonally dominant if

   (A) $|a_{ii}| \geq \sum_{\substack{j=1 \\ i\neq j}}^{n} |a_{ij}|, \ i = 1,2,...,n$

   (B) $|a_{ii}| \geq \sum_{\substack{j=1 \\ i\neq j}}^{n} |a_{ij}|, \ i = 1,2,...,n$ and $|a_{ii}| > \sum_{\substack{j=1 \\ i\neq j}}^{n} |a_{ij}|$, for any $i = 1,2,...,n$

   (C) $|a_{ii}| \geq \sum_{j=1}^{n} |a_{ij}|, \ i = 1,2,...,n$ and $|a_{ii}| > \sum_{j=1}^{n} |a_{ij}|$, for any $i = 1,2,...,n$

   (D) $|a_{ii}| \geq \sum_{j=1}^{n} |a_{ij}|, \ i = 1,2,...,n$

2. Using $[x_1, x_2, x_3] = [1,3,5]$ as the initial guess, the values of $[x_1, x_2, x_3]$ after three iterations in the Gauss-Seidel method for

$$\begin{bmatrix} 12 & 7 & 3 \\ 1 & 5 & 1 \\ 2 & 7 & -11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ 6 \end{bmatrix}$$

   are

   (A) [-2.8333 -1.4333 -1.9727]
   (B) [1.4959 -0.90464 -0.84914]
   (C) [0.90666 -1.0115 -1.0243]
   (D) [1.2148 -0.72060 -0.82451]

3. To ensure that the following system of equations,

$$\begin{aligned} 2x_1 + 7x_2 - 11x_3 &= 6 \\ x_1 + 2x_2 + x_3 &= -5 \\ 7x_1 + 5x_2 + 2x_3 &= 17 \end{aligned}$$

converges using the Gauss-Seidel method, one can rewrite the above equations as follows:

(A) $\begin{bmatrix} 2 & 7 & -11 \\ 1 & 2 & 1 \\ 7 & 5 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -5 \\ 17 \end{bmatrix}$

(B) $\begin{bmatrix} 7 & 5 & 2 \\ 1 & 2 & 1 \\ 2 & 7 & -11 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 17 \\ -5 \\ 6 \end{bmatrix}$

(C) $\begin{bmatrix} 7 & 5 & 2 \\ 1 & 2 & 1 \\ 2 & 7 & -11 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -5 \\ 17 \end{bmatrix}$

(D) The equations cannot be rewritten in a form to ensure convergence.

4. For $\begin{bmatrix} 12 & 7 & 3 \\ 1 & 5 & 1 \\ 2 & 7 & -11 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 22 \\ 7 \\ -2 \end{bmatrix}$ and using $\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$ as the initial guess, the values of $\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$ are found at the end of each iteration as

Iteration #	x_1	x_2	x_3
1	0.41667	1.1167	0.96818
2	0.93990	1.0184	1.0008
3	0.98908	1.0020	0.99931
4	0.99899	1.0003	1.0000

At what first iteration number would you trust at least 1 significant digit in your solution?

(A) 1
(B) 2
(C) 3
(D) 4

5. The algorithm for the Gauss-Seidel method to solve $[A][X]=[C]$ is given as follows when using $n\max$ iterations. The initial value of $[X]$ is stored in $[X]$.

(A) Sub Seidel $(n, a, x, rhs, \text{nmax})$

```
For k = 1 To nmax
For i = 1 To n
For j = 1 To n
If (i <> j) Then
Sum = Sum + a(i, j) * x(j)
endif
Next j
x(i) = (rhs(i) – Sum) / a(i,i)
Next i
Next j
End Sub
```

(B) Sub Seidel $(n, a, x, rhs, \text{nmax})$

```
For k = 1 To nmax
  For i = 1 To n
  Sum = 0
  For j = 1 To n
  If (i <> j) Then
  Sum = Sum + a(i, j) * x(j)
  endif
  Next j
  x(i) = (rhs(i) – Sum) / a(i,i)
  Next i
  Next k
  End Sub
```

(C) Sub Seidel $(n, a, x, rhs, \text{nmax})$

```
For k = 1To nmax
For i = 1To n
Sum = 0
For j = 1To n
Sum = Sum + a(i, j) * x(j)
Next j
x(i) = (rhs(i) – Sum) / a(i,i)
Next i
```

```
Next k
End Sub
```

(D)
```
Sub Seidel(n, a, x, rhs, nmax)
For k = 1 To nmax
For i = 1 To n
Sum = 0
For j = 1 To n
If (i <> j) Then
Sum = Sum + a(i, j) * x(j)
endif
Next j
x(i) = (rhs(i) - Sum) / a(i,i)
Next i
Next k
End Sub
```

6. Thermistors measure temperature, have a nonlinear output and are valued for a limited range. So when a thermistor is manufactured, the manufacturer supplies a resistance vs. temperature curve. An accurate representation of the curve is generally given by

$$\frac{1}{T} = a_0 + a_1 \ln(R) + a_2 \{\ln(R)\}^2 + a_3 \{\ln(R)\}^3$$

where T is temperature in Kelvin, R is resistance in ohms, and a_0, a_1, a_2, a_3 are constants of the calibration curve. Given the following for a thermistor

R	T
ohm	°C
1101.0	25.113
911.3	30.131
636.0	40.120
451.1	50.128

the value of temperature in °C for a measured resistance of 900 ohms most nearly is

(A) 30.002
(B) 30.473
(C) 31.272
(D) 31.445

<u>For a complete solution, refer to the links at the end of the book.</u>