Holistic Numerical Methods Institute
committed to bringing numerical methods to undergraduates

Multiple-Choice Test
Newton Raphson Method
Nonlinear Equations
COMPLETE SOLUTION SET

1. The Newton-Raphson method of finding roots of nonlinear equations falls under the category of _____________ methods.
   (A) bracketing
   (B) open
   (C) random
   (D) graphical

Solution
The correct answer is (B).

The Newton-Raphson method is an open method since the guess of the root that is needed to get the iterative method started is a single point. Other open methods such as the secant method use two initial guesses of the root, but they do not have to bracket the root.

2. The Newton-Raphson method formula for finding the square root of a real number R from the equation $x^2 - R = 0$ is,

(A) $x_{i+1} = \frac{x_i}{2}$

(B) $x_{i+1} = \frac{3x_i}{2}$

(C) $x_{i+1} = \frac{1}{2}\left(x_i + \frac{R}{x_i}\right)$

(D) $x_{i+1} = \frac{1}{2}\left(3x_i - \frac{R}{x_i}\right)$

Solution

The correct answer is (C).

The Newton-Raphson method formula for solving $f(x) = 0$ is

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

where

$$f(x) = x^2 - R$$
$$f'(x) = 2x$$

Thus,

$$\begin{aligned}
x_{i+1} &= x_i - \frac{x_i^2 - R}{2x_i} \\
&= x_i - \left(\frac{x_i^2 - R}{2x_i}\right) \\
&= x_i - \left(\frac{x_i}{2} - \frac{R}{2x_i}\right) \\
&= x_i - \frac{x_i}{2} + \frac{R}{2x_i} \\
&= \frac{1}{2}x_i + \frac{R}{2x_i} \\
&= \frac{1}{2}\left(x_i + \frac{R}{x_i}\right)
\end{aligned}$$

3. The next iterative value of the root of $x^2 - 4 = 0$ using the Newton-Raphson method, if the initial guess is 3, is

(A) 1.5
(B) 2.067
(C) 2.167
(D) 3.000

Solution
The correct answer is (C).

The estimate of the root is

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Chose $i = 0$,

$$x_0 = 3$$

$$f(x_0) = {x_0}^2 - 4$$
$$= 3^2 - 4$$
$$= 5$$

$$f'(x_0) = 2x_0$$
$$= 2 \times 3$$
$$= 6$$

Thus,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$
$$= 3 - \frac{5}{6}$$
$$= 2.166$$

4. The root of the equation $f(x) = 0$ is found by using the Newton-Raphson method. The initial estimate of the root is $x_0 = 3$, $f(3) = 5$. The angle the line tangent to the function $f(x)$ makes at $x = 3$ is $57°$ with respect to the x-axis. The next estimate of the root, x_1 most nearly is

(A) –3.2470

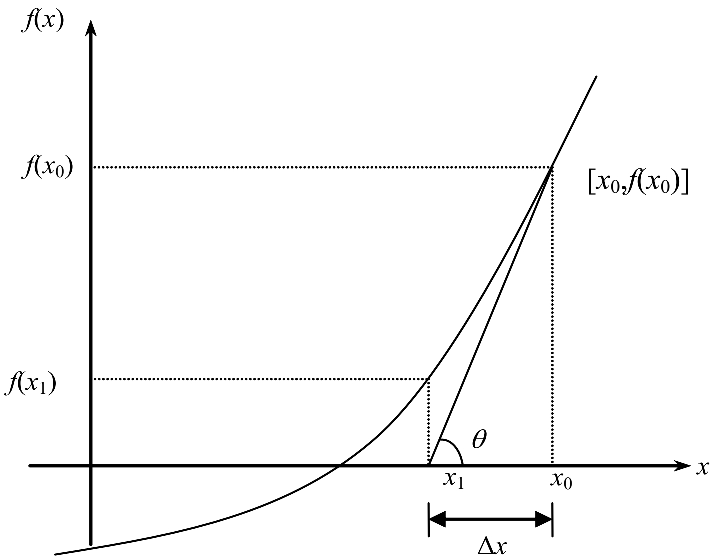
(B) −0.24704

(C) 3.2470

(D) 6.2470

Solution

The correct answer is (B).



Since,

$$\theta = 57°,$$
$$x_0 = 3, \text{ and}$$
$$f(x_0) = 5$$
$$tan(\theta) = \frac{rise}{run}$$
$$tan(57°) = \frac{f(x_0) - f(x_1)}{x_0 - x_1}$$
$$tan(57°) = \frac{5 - 0}{3 - x_1}$$
$$x_1 = \frac{5 - 3(tan(57°))}{-tan(57°)}$$
$$= \frac{5 - 3 \times 1.5399}{-1.5399}$$

$$= -0.24704$$

5. The root of $x^3 = 4$ is found by using the Newton-Raphson method. The successive iterative values of the root are given in the table below.

Iteration Number	Value of Root
0	2.0000
1	1.6667
2	1.5911
3	1.5874
4	1.5874

The iteration number at which I would first trust at least two significant digits in the answer is

(A) 1
(B) 2
(C) 3
(D) 4

Solution
The correct answer is (C).

The absolute relative approximate error for the first iteration is

$$|\in_a| = \left|\frac{1.6667 - 2}{1.6667}\right| \times 100$$
$$= |-0.19999| \times 100$$
$$= 19.999\%$$

Since, $19.99\% \leq 0.5 \times 10^{2-m}$, $m = 0$. There are no significant digits correct.
The absolute relative approximate error for the second iteration is

$$|\in_a| = \left|\frac{1.5911 - 1.6667}{1.5911}\right| \times 100$$
$$= |-0.047514| \times 100$$
$$= 4.7514\%$$

Since, $4.7514\% \leq 0.5 \times 10^{2-m}$, $m = 1$. There is at least one significant digit correct.
The absolute relative approximate error for the third iteration is

$$|\in_a| = \left|\frac{1.5874 - 1.5911}{1.5874}\right| \times 100$$
$$= |-0.0023308| \times 100$$
$$= 0.23308\%$$

Since, $0.23308\% \leq 0.5 \times 10^{2-m}$,$m = 2$. There are at least two significant digits correct.
Thus, the third iteration is the first iteration in which at least two significant digits are correct.

6. The ideal gas law is given by

$$pv = RT$$

where p is the pressure, v is the specific volume, R is the universal gas constant, and T is the absolute temperature. This equation is only accurate for a limited range of pressure and temperature. Vander Waals came up with an equation that was accurate for larger range of pressure and temperature given by

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

where a and b are empirical constants dependent on a particular gas. Given the value of $R = 0.08$, $a = 3.592$, $b = 0.04267$, $p = 10$ and $T = 300$ (assume all units are consistent), one is going to find the specific volume, v, for the above values. Without finding the solution from the Vander Waals equation, what would be a good initial guess for v?

(A) 0
(B) 1.2
(C) 2.4
(D) 3.6

Solution
The correct answer is (C).

From the physics of the problem, the initial guess can be found from the linear relationship of the ideal gas law

$$pv = RT$$

$$v = \frac{RT}{p}$$

$$= \frac{0.08 \times 300}{10}$$

$$= 2.4$$