Multiple-Choice Test

Chapter 02.02
Differentiation of Continuous Functions

1. The definition of the first derivative of a function \( f(x) \) is
   \[
   f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) + f(x)}{\Delta x} \\
   \text{(A)} \quad f'(x) = \frac{f(x + \Delta x) + f(x)}{\Delta x} \\
   \text{(B)} \quad f'(x) = \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
   \text{(C)} \quad f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) + f(x)}{\Delta x} \\
   \text{(D)} \quad f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}
   \]

2. The exact derivative of \( f(x) = x^3 \) at \( x = 5 \) is most nearly
   (A) 25.00
   (B) 75.00
   (C) 106.25
   (D) 125.00

3. Using the forwarded divided difference approximation with a step size of 0.2, the derivative of \( f(x) = 5e^{2.3x} \) at \( x = 1.25 \) is
   (A) 163.4
   (B) 203.8
   (C) 211.1
   (D) 258.8

4. A student finds the numerical value of \( \frac{d}{dx} (e^x) = 20.220 \) at \( x = 3 \) using a step size of 0.2. Which of the following methods did the student use to conduct the differentiation?
   (A) Backward divided difference
   (B) Calculus, that is, exact
   (C) Central divided difference
   (D) Forward divided difference
5. Using the backward divided difference approximation, \( \frac{d}{dx} (e^x) = 4.3715 \) at \( x = 1.5 \) for a step size of 0.05. If you keep halving the step size to find \( \frac{d}{dx} (e^x) \) at \( x = 1.5 \) before two significant digits can be considered to be at least correct in your answer, the step size would be (you cannot use the exact value to determine the answer)
   (A) 0.05/2
   (B) 0.05/4
   (C) 0.05/8
   (D) 0.05/16

6. The heat transfer rate \( q \) over a surface is given by
   \[
   q = -kA \frac{dT}{dy}
   \]
   where
   \( k = \) thermal conductivity \( \left( \frac{J}{s \cdot m \cdot K} \right) \)
   \( A = \) surface area \( \left( m^2 \right) \)
   \( T = \) temperature \( (K) \)
   \( y = \) distance normal to the surface \( (m) \)

Given
   \( k = 0.025 \frac{J}{s \cdot m \cdot K} \)
   \( A = 3 \text{ m}^2 \)

the temperature \( T \) over the surface varies as
   \( T = -1493y^3 + 2200y^2 - 1076y + 500 \)

The heat transfer rate \( q \) at the surface most nearly is
   (A) -1076 W
   (B) 37.5 W
   (C) 80.7 W
   (D) 500 W

For a complete solution, refer to the links at the end of the book.