Holistic Numerical Methods Institute
committed to bringing numerical methods to undergraduates

Multiple-Choice Test
Background
Differentiation
COMPLETE SOLUTION SET

1. The definition of the first derivative of a function $f(x)$ is
   (A) $f'(x) = \dfrac{f(x+\Delta x) + f(x)}{\Delta x}$
   (B) $f'(x) = \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$
   (C) $f'(x) = \lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x) + f(x)}{\Delta x}$
   (D) $f'(x) = \lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

Solution
The correct answer is (D).
The definition of the first derivative of the function $f(x)$ is

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Choice (B) is incorrect as it is an approximate method to calculate the first derivative of a function $f(x)$. In fact, choice (B) is the forward divided difference method of approximately calculating the first derivative of a function.

2. Given $y = 5e^{3x} + \sin x$, $\frac{dy}{dx}$ is

(A) $5e^{3x} + \cos x$
(B) $15e^{3x} + \cos x$
(C) $15e^{3x} - \cos x$
(D) $2.666e^{3x} - \cos x$

Solution
The correct answer is (B)

Use the sum rule of differentiation

$$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$$

Re-write the function as

$$y = u + v$$

where

$$u = 5e^{3x}$$
$$v = \sin x$$

Find $\frac{du}{dx}$ and $\frac{dv}{dx}$

$$\frac{d}{dx}(5e^{3x}) = 15e^{3x} \qquad \left(\frac{d}{dx}(e^{ax}) = ae^{ax}\right)$$

$$\frac{d}{dx}(\sin x) = \cos x \qquad \left(\frac{d}{dx}(\sin x) = \cos x\right)$$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$
$$= 15e^{3x} + \cos x$$

3. Given $y = \sin 2x$, $\frac{dy}{dx}$ at $x = 3$ is most nearly

   (A) 0.9600
   (B) 0.9945
   (C) 1.920
   (D) 1.989

Solution

The correct answer is (C).

Using the chain rule

$$u = 2x$$

$$y = \sin u$$

$$\frac{du}{dx} = 2$$

$$\frac{dy}{du} = \cos u$$

$$= \cos 2x$$

Since,

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = (\cos 2x) \times 2$$

At $x = 3$

$$\frac{dy}{dx} = (\cos 6) \times 2$$ (Don't forget to use radians)

$$= 0.9602 \times 2$$

$$= 1.920$$

4. Given $y = x^3 \ln x$, $\frac{dy}{dx}$ is

   (A) $3x^2 \ln x$
   (B) $3x^2 \ln x + x^2$
   (C) x^2
   (D) $3x$

Solution

The correct answer is (B).

Using the product rule,

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx} \tag{1}$$

$$u = x^3$$

$$v = \ln x$$

$$\frac{du}{dx} = 3x^2$$

$$\frac{dv}{dx} = \frac{1}{x}$$

Substituting into Equation (1),

$$\frac{dy}{dx} = x^3 \times \frac{1}{x} + \ln x \times 3x^2$$

$$= x^2 + 3x^2 \ln x$$

5. The velocity of a body as a function of time is given as $v(t) = 5e^{-2t} + 4$, where t is in seconds, and v is in m/s. The acceleration in m/s^2 at $t = 0.6$ s is

(A) -3.012
(B) 5.506
(C) 4.147
(D) -10.00

Solution

The correct answer is (A)

$$
\begin{aligned}
a(t) &= \frac{dv}{dt} \\
&= \frac{d}{dt}\left(5e^{-2t} + 4\right) \\
&= -10e^{-2t}
\end{aligned}
\qquad \left(\frac{d}{dx}(e^{ax}) = ae^{ax}\right)
$$

$$
\begin{aligned}
a(0.6) &= -10e^{-2(0.6)} \\
&= -3.012 \ \text{m/s}^2
\end{aligned}
$$

6. If $x^2 + 2xy = y^2$, then $\frac{dy}{dx}$ is

(A) $\frac{x+y}{y-x}$
(B) $2x+2y$
(C) $\frac{x+1}{y}$
(D) $-x$

Solution
The correct answer is (A).

$$\frac{d}{dx}[x^2] + \frac{d}{dx}[2xy] = \frac{d}{dx}[y^2]$$

$$2x + 2x\frac{dy}{dx} + 2y = 2y\frac{dy}{dx} \qquad \left(\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}\right)$$

$$x + y = (y-x)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{x+y}{y-x}$$