Abstract: This simulation illustrates the Newton divided difference method of interpolation. Given n data points of y versus x, you are then required to find the value of y at a particular value of x using first, second, and third order interpolation. So one has to first pick the needed data points, and then use those to interpolate the data.

INPUTS: Enter the following

Array of x data:

\[
\begin{pmatrix}
2.0 \\
4.25 \\
5.25 \\
7.81 \\
9.20 \\
10.60
\end{pmatrix}
\]

Array of y data

\[
\begin{pmatrix}
7.2 \\
7.1 \\
6.0 \\
5.0 \\
3.5 \\
5.0
\end{pmatrix}
\]

Value of x at which y is desired: \(x_{\text{desired}} := 4\)

SOLUTION

This function considers the x and y data and selects the two points closest data points that bracket the desired value of x.

\[
\text{firsttwo := n} \leftarrow \text{rows(x)} \\
\text{comp} \leftarrow \left| x - x_{\text{desired}} \right| \\
c \leftarrow \text{min(comp)} \\
\text{for } i \in 0..n - 1 \\
ci \leftarrow i \text{ if } \text{comp}_i = c
\]
If more than two values are desired, the following function selects the subsequent values and puts all the values into a matrix, maintaining the original data order.

\[ \text{selectxy(num)} := \begin{align*} 
& n \leftarrow \text{rows}(x) \\
& \text{comp} \leftarrow \left\lfloor x - x_{\text{desired}} \right\rfloor \\
& \text{for } i \in 0..n - 1 \\
& A_{i,1} \leftarrow i \\
& A_{i,0} \leftarrow \text{comp}_i \\
& A \leftarrow \text{csort}(A,0) \\
& \text{for } i \in 0..n - 1 \\
& A_{i,2} \leftarrow i \\
& A \leftarrow \text{csort}(A,1) \\
& A \leftarrow A^{(2)} 
\end{align*} \]
These two functions use the above functions to assign the selected data to new variables.

\[ x_{\text{sub}}(n) := \text{submatrix}(\text{selectxy}(n), 0, \text{rows}(\text{selectxy}(n)) - 1, 0, 0) \]

\[ y_{\text{sub}}(n) := \text{submatrix}(\text{selectxy}(n), 0, \text{rows}(\text{selectxy}(n)) - 1, 1, 1) \]

**Given y versus x data points**

![Graph of y versus x data points](image_url)
Linear interpolation (first order polynomial)

Choose two data points

\[ x_s := x_{sub}^{(2)} \]

\[ y_s := y_{sub}^{(2)} \]

\[ x_s = \begin{pmatrix} 2 \\ 4.25 \end{pmatrix} \]

\[ y_s = \begin{pmatrix} 7.2 \\ 7.1 \end{pmatrix} \]

Calculating coefficients of Newton's Divided difference polynomial

\[ b_0 := y_0 \]

\[ b_0 = 7.2 \]

\[ b_1 := \frac{y_1 - y_0}{x_1 - x_0} \]

\[ b_1 = -0.044 \]

Newton's divided difference formula for linear interpolation

\[ f_1(x) := b_0 + b_1(x - x_0) \]

Calculating value at desired point

\[ f_1(x_{desired}) = 7.111 \]

\[ \text{results}_{0,0} := f_1(x_{desired}) \]

\[ \text{range} := \min(x_s), \min(x_s) + \frac{\max(x_s) - \min(x_s)}{1000} \cdot \max(x_s) \]
Linear interpolation

\[ f_1(x_{\text{desired}}) \]

\[ x_{\text{range}}, x_{\text{desired}} \]
Quadratic interpolation (second order polynomial)

Pick three data points

\[ x_s := x_{\text{sub}(3)} \]
\[ y_s := y_{\text{sub}(3)} \]

\[ x_s = \begin{pmatrix} 2 \\ 4.25 \\ 5.25 \end{pmatrix} \]
\[ y_s = \begin{pmatrix} 7.2 \\ 7.1 \\ 6 \end{pmatrix} \]

The general formula for quadratic interpolation using Newton's Divided Difference is as follows:

\[ y = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) \]

where \( b_1 \) represents the first divided difference, \( b_2 \) represents the second divided difference and so on.

\[ b_0 := y_0 \]
\[ b_0 = 7.2 \]

\[ b_1 := \frac{y_1 - y_0}{x_1 - x_0} \]
\[ b_1 = -0.044 \]

\[ b_2 := \frac{\left( \frac{y_2 - y_1}{x_2 - x_1} - \frac{y_1 - y_0}{x_1 - x_0} \right)}{x_2 - x_0} \]
\[ b_2 = -0.325 \]

\[ f_{\text{prev}} := f_1(x_{\text{desired}}) \]
\[ f_2(x) := b_0 + b_1(x - x_{s_0}) + b_2(x - x_{s_0})(x - x_{s_1}) \]

**Calculating value at desired point**

\[ f_2(x_{\text{desired}}) = 7.274 \]

\[ f_{\text{new}} := f_2(x_{\text{desired}}) \]

\[ \text{results}_{0, 1} := f_2(x_{\text{desired}}) \]

**Absolute relative approximate error**

\[ \varepsilon_a := \left| \frac{f_2(x_{\text{desired}}) - f_1(x_{\text{desired}})}{f_2(x_{\text{desired}})} \right| \cdot 100 \]

\[ \varepsilon_a = 2.233 \]

\[ \text{results}_{1, 1} := \varepsilon_a \]

**Number of significant digits at least correct in the solution**

\[ \text{sigdigits} := \begin{cases} 0 & \text{if } |\varepsilon_a| \leq 0 \\ \text{trunc} \left( 2 - \log \left( \frac{|\varepsilon_a|}{0.5} \right) \right) & \text{otherwise} \end{cases} \]

\[ \text{sigdigits} = 1 \]

\[ \text{results}_{2, 1} := \text{sigdigits} \]

\[ \text{range} := \min(x_s), \min(x_s) + \frac{\max(x_s) - \min(x_s)}{1000} \ldots \max(x_s) \]
Quadratic interpolation

$y_s$

$f_2(\text{range})$

$f_2(x_{\text{desired}})$

$\times \times$

$x_s, \text{range}, x_{\text{desired}}$

$x, \text{range}, x_{\text{desired}}$
Cubic interpolation (third order polynomial)

Pick four data points

\[ x_s := x_{\text{sub}(4)} \]

\[ y_s := y_{\text{sub}(4)} \]

\[
\begin{bmatrix}
2 \\
4.25 \\
5.25 \\
7.81
\end{bmatrix}
\]

\[
\begin{bmatrix}
7.2 \\
7.1 \\
6 \\
5
\end{bmatrix}
\]

Calculating coefficients of Newton's divided difference polynomial

\[ b_0 := y_{s_0} \]

\[ b_0 = 7.2 \]

\[ b_1 := \frac{y_{s_1} - y_{s_0}}{x_{s_1} - x_{s_0}} \]

\[ b_1 = -0.044 \]

\[ b_2 := \frac{\left( \frac{y_{s_2} - y_{s_1}}{x_{s_2} - x_{s_1}} - b_1 \right)}{x_{s_2} - x_{s_0}} \]

\[ b_2 = -0.325 \]

\[ b_3 := \frac{\left( \frac{y_{s_3} - y_{s_2}}{x_{s_3} - x_{s_2}} - \frac{y_{s_2} - y_{s_1}}{x_{s_2} - x_{s_1}} \right) - b_2}{x_{s_3} - x_{s_0}} \]

\[ b_3 = 0.09 \]
\( f_{\text{prev}} := f_2(x_{\text{desired}}) \)

\[
 f_3(x) := b_0 + b_1(x-x_{s_0}) + b_2(x-x_{s_0})(x-x_{s_1}) + b_3(x-x_{s_0})(x-x_{s_1})(x-x_{s_2})
\]

**Value of function at desired point**

\[
f_3(x_{\text{desired}}) = 7.33
\]

\[
 f_{\text{new}} := f_3(x_{\text{desired}})
\]

\[
 \text{results}_{0,2} := f_3(x_{\text{desired}})
\]

**Absolute relative approximate error**

\[
 \varepsilon_{a} := \left| \frac{f_3(x_{\text{desired}}) - f_2(x_{\text{desired}})}{f_3(x_{\text{desired}})} \right| \cdot 100
\]

\[
 \varepsilon_{a} = 0.769
\]

\[
 \text{results}_{1,2} := \varepsilon_{a}
\]

**Number of significant digits at least correct in the solution**

\[
 \text{sigdigits} := \begin{cases} 0 & \text{if } |\varepsilon_{a}| \leq 0 \\ \text{trunc} \left( 2 - \log \left( \left| \varepsilon_{a} \right| / 0.5 \right) \right) & \text{otherwise} \end{cases}
\]

\[
 \text{sigdigits} = 1
\]

\[
 \text{results}_{2,2} := \text{sigdigits}
\]

\[
 \text{range} := \min(x_s), \min(x_s) + \frac{\max(x_s) - \min(x_s)}{1000} \ldots \max(x_s)
\]
Cubic interpolation

\[ y \]

\[ f_3(\text{range}) \]

\[ f_3(x_{\text{desired}}) \]

\[ x, \text{ range}, x_{\text{desired}} \]
### Summary of Newton’s Divided Difference Method of Interpolation

<table>
<thead>
<tr>
<th></th>
<th>First Order</th>
<th>Second Order</th>
<th>Third Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interpolated Value</td>
<td>7.11111</td>
<td>7.2735</td>
<td>7.32988</td>
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<tr>
<td>Absolute relative approximate error</td>
<td>0</td>
<td>2.23267</td>
<td>0.76909</td>
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<tr>
<td>Number of significant digits</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>