INPUTS: Enter the following

Function in \( f(x) = 0 \)
\[ f(x) := x^3 - 0.03 \cdot x^2 + 2.4 \cdot 10^{-6} \]

Range of \( x \) you want to see the function
\[ x := 0, .001 .. .03 \]

Upper guess
\[ x_{\text{guess}1} := 0.02 \]

Lower guess
\[ x_{\text{guess}2} := 0.01 \]

Maximum number of iterations
\[ n_{\text{max}} := 7 \]

Initial guess for Mathcad solution
\[ x_{\text{guess}} := 0.01 \]

SOLUTION

Entered function at given interval
Exact Solution:

This is the true solution found by Mathcad.

\[ x := x_{\text{guess}} \]

\[ x_{\text{true}} := \text{root}(f(x), x) \]

\[ x_{\text{true}} = 0.01133 \]

Here the secant method algorithm is applied to generate the estimated root, true error, absolute relative true error, absolute approximate error, absolute relative approximate error, and the number of significant digits at least correct in the estimated root as a function of number of iterations.

\[
x_r(n) := \\
\quad i \leftarrow 1 \\
\quad x_1 \leftarrow x_{\text{guess}1} \\
\quad x_2 \leftarrow x_{\text{guess}2} \\
\quad \text{while } i \leq n \\
\quad \quad x_{\text{root}} \leftarrow x_2 - \frac{[f(x_2)(x_1 - x_2)]}{f(x_1) - f(x_2)} \\
\quad \quad x_1 \leftarrow x_2 \\
\quad \quad x_2 \leftarrow x_{\text{root}} \\
\quad \quad i \leftarrow i + 1 \\
\quad x_{\text{root}} \\
\]

\[ n := 1 \ldots n_{\text{max}} \]

True error:

\[ E_{t}(n) := x_{\text{true}} - x_r(n) \]

Absolute relative true error:

\[ \varepsilon_{t}(n) := \left| \frac{E_t(n)}{x_{\text{true}}} \right| \times 100 \]
Absolute approximate error:
\[ E_a(n) := x_r(n) - x_r(n - 1) \]

Absolute relative approximate error:
\[ \varepsilon_a(n) := \begin{cases} 0 & \text{if } n \leq 1 \\ \left( \frac{|E_a(n)|}{x_r(n)} \right) \cdot 100 & \text{otherwise} \end{cases} \]

Significant digits at least correct:
\[ \text{sigdigits}(n) := \begin{cases} 0 & \text{if } |\varepsilon_a(n)| \leq 0 \\ 2 - \log\left( \left| \varepsilon_a(n) \right| \left| \frac{\varepsilon_a(n)}{0.5} \right| \right) & \text{otherwise} \end{cases} \]
## Table of Values:

<table>
<thead>
<tr>
<th>n =</th>
<th>( x_r(n) = )</th>
<th>( E_t(n) = )</th>
<th>( \varepsilon_t(n) = )</th>
<th>( E_d(n) = )</th>
<th>( \varepsilon_d(n) = )</th>
<th>( \text{trunc(sigdigits(n))} = )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.012</td>
<td>-6.66666 \cdot 10^{-4}</td>
<td>5.88235</td>
<td>0.012</td>
<td>0</td>
<td>0</td>
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<td>-1.80176 \cdot 10^{-5}</td>
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<td>-7.93893 \cdot 10^{-6}</td>
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<td>-8.04469 \cdot 10^{-6}</td>
<td>0.07098</td>
<td>1.05766 \cdot 10^{-7}</td>
<td>9.32568 \cdot 10^{-4}</td>
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<td>0.07098</td>
<td>-1.44433 \cdot 10^{-11}</td>
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<td>0.07098</td>
<td>0</td>
<td>1.68251 \cdot 10^{-13}</td>
<td>14</td>
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<tr>
<td>7</td>
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<td>-8.04468 \cdot 10^{-6}</td>
<td>0.07098</td>
<td>0</td>
<td>3.05911 \cdot 10^{-14}</td>
<td>15</td>
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</table>
Estimated root as a function of number of iterations
True error as a function of number of iterations

Number of iterations

True error

$E_t(n)$
Absolute relative true error as a function of number of iterations
Approximate error as a function of number of iterations

![Graph showing approximate error as a function of number of iterations](image)
Absolute relative approximate error as a function of number of iterations

\[ |e_a(n)| \]

Number of iterations

<table>
<thead>
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<th>n</th>
</tr>
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<tr>
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</tr>
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<td>3</td>
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<tr>
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<td>6</td>
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<tr>
<td>7</td>
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</tbody>
</table>
Least number of significant digits at least correct as a function of number of iterations

\begin{align*}
\text{Significant digits} \quad \text{trunc(sigdigits(n))}
\end{align*}