Chapter 03.00B

Physical Problem for Nonlinear Equations Chemical Engineering

Problem Statement

Years ago, a businessperson called me and wanted to know how he could find how much oil was left in his storage tank. His tank was spherical and was 6 feet in diameter. Well, I suggested him to get a 8ft steel ruler and use it as a dipstick (Figure 1). Knowing the height

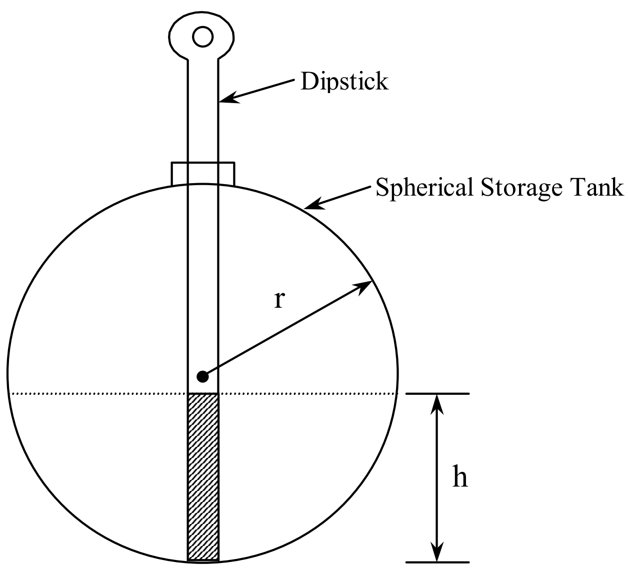

Figure 1 Oil in a spherical storage tank.

to which the dip-stick would become wet with oil, one would know the height h of the oil in the tank. The volume V of oil left in the tank then is

$$V = \frac{\pi h^2 (3r - h)}{3} \tag{1}$$

where, r is the radius of the tank. But, he did not stop there. He wanted me to design a steel ruler for him so that he would directly get the reading from the dipstick. How would I design such a dipstick?

Solution

The problem is inverse of what he wanted originally. To design a dipstick, I would have to mark the height corresponding to a volume. To do that I would need to solve the equation

$$V = \frac{\pi h^2(3r-h)}{3}$$

for the height for a given volume and radius. For example, where would you mark the scale for $4\,ft^3$ of oil?

$$4 = \frac{\pi h^2(3\times 3-h)}{3}$$

$$h^3 - 9h^2 + \frac{12}{\pi} = 0$$

$$f(h) = h^3 - 9h^2 + 3.8197 = 0$$

Therefore, this nonlinear equation needs to be solved. To mark the scale for other volumes, you will need to substitute the value for the volume and solve for h. Continue to do this for different preset volumes to develop the scale.

Appendix A: Derivation of the formula for the volume of the oil based on radius of the tank and the height of oil in the tank.

How do you find that the volume of oil is given by

$$V = \frac{\pi h^2(3r-h)}{3}$$

From calculus,

$$V = \int_{r-h}^{r} A\,dx$$

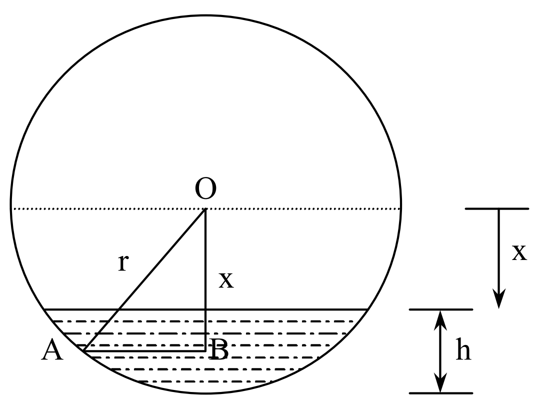

Figure 2 Deriving the equation for volume of oil in the tank.

where A is the cross-sectioned area at the location x, from the center of the sphere. The lower limit of integration is $x = r-h$ as that is where the oil line is and the upper limit is r as that is the bottom of the sphere. So, what is A at any location x?

From Figure 2, for a location x

$$OB = x$$
$$OA = r$$

then

$$AB = \sqrt{OA^2 - OB^2}$$
$$= \sqrt{r^2 - x^2}$$

and AB is the radius of the area at x. So the area at location x is

$$A = \pi (AB)^2$$
$$= \pi\left(r^2 - x^2\right)$$

so

$$V = \int_{r-h}^{r} \pi\left(r^2 - x^2\right)dx$$
$$= \pi\left(r^2 x - \frac{x^3}{3}\right)_{r-h}^{r}$$
$$= \pi\left[\left(r^2 r - \frac{r^3}{3}\right) - \left(r^2(r-h) - \frac{(r-h)^3}{3}\right)\right]$$
$$= \frac{\pi h^2(3r-h)}{3}.$$

NONLINEAR EQUATIONS	
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