### Problem Statement
The general form of Taylor series is given below.

\[
f(x + h) = f(x) + f'(x)h + f''(x)\frac{h^2}{2!} + f'''(x)\frac{h^3}{3!} + f^{(4)}(x)\frac{h^4}{4!} + f^{(5)}(x)\frac{h^5}{5!} + \ldots
\]

Given
\[
f(4) = 0 \quad f'(4) = 7 \quad f''(4) = 10 \quad f'''(4) = 30\]

and all other higher derivatives of \( f(x) \) at \( x = 4 \) are zero. The function \( f(x) \) is of polynomial form,

\[
f(x) = a_0 + a_1x + a_2x^2 + a_3x^3.
\]

Find the function \( f(x) \).

### Solution
Choosing \( x = 4 \)

\[
f(4 + h) = f(4) + f'(4)h + f''(4)\frac{h^2}{2!} + f'''(4)\frac{h^3}{3!}
\]
as all other higher derivatives are zero.

Substituting the given values of the function and its derivatives at \( x=4 \) gives

\[
f(4 + h) = 0 + 7h + 10\frac{h^2}{2!} + 30\frac{h^3}{3!}
\]

\[
= 7h + 5h^2 + 5h^3
\]

Now substitute

\[
x = 4 + h
\]
on the left hand side, and hence

\[
h = x - 4
\]
on the right hand side, we get

\[
f(x) = 7(x - 4) + 5(x - 4)^2 + 5(x - 4)^3
\]

\[
= 7(x - 4) + 5(x^2 + 16 - 8x)
\]

\[
+ 5(x^3 - 64 - 12x^2 + 48x)
\]

\[
= 5x^3 - 55x^2 + 207x - 268
\]

### Questions
Confirm the given values of the function and its derivatives at \( x=4 \).

### References
Taylor Series Revisited,
http://numericalmethods.eng.usf.edu/topics/taylor_series.html