Problem Statement

The general form of Taylor series is given below.

$$f(x+h)=f(x)+f'(x)h+f''(x)\frac{h^2}{2!}$$
$$+f'''(x)\frac{h^3}{3!}+f''''(x)\frac{h^4}{4!}$$
$$+f'''''(x)\frac{h^5}{5!}+.....$$

Given

$f(4)=0$ $f'(4)=7$ $f''(4)=10$ $f'''(4)=30$ and all other higher derivatives of $f(x)$ at $x=4$ are zero. The function $f(x)$ is of polynomial form,

$$f(x)=a_0+a_1x+a_2x^2+a_3x^3.$$

Find the function $f(x)$.

Solution

Choosing $x=4$

$$f(4+h)=f(4)+f'(4)h+f''(4)\frac{h^2}{2!}+f'''(4)\frac{h^3}{3!}$$

as all other higher derivatives are zero.

Substituting the given values of the function and its derivatives at x=4 gives

$$f(4+h)=0+7h+10\frac{h^2}{2!}+30\frac{h^3}{3!}$$
$$=7h+5h^2+5h^3$$

Now substitute

$$x=4+h$$

on the left hand side, and hence

$$h=x-4$$

on the right hand side, we get

$$f(x)=7(x-4)+5(x-4)^2+5(x-4)^3$$
$$=7(x-4)+5(x^2+16-8x)$$
$$+5(x^3-64-12x^2+48x)$$
$$=5x^3-55x^2+207x-268$$

Questions

Confirm the given values of the function and its derivatives at x=4.

References

Taylor Series Revisited,
http://numericalmethods.eng.usf.edu/topics/taylor_series.html