Clearing up the confusion about diagonally dominant matrices – Part 3

In this blog, we clarify the definition of a strictly diagonally dominant matrix.

What is a strictly diagonally dominant matrix?

A $n \times n$ square matrix $[A]$ is a diagonally dominant matrix if

$$\left|a_{ii}\right| > \sum_{\substack{j=1 \\ i \neq j}}^{n} | a_{ij} | \text{ for } i = 1,2,.....,n$$

that is, for each row, the absolute value of the diagonal element is strictly greater than the sum of the absolute values of the rest of the elements of that row.

Example

Give examples of strictly diagonally dominant matrices and not strictly diagonally dominant matrices.

Solution

The matrix

$$[A] = \begin{bmatrix} 15 & 6 & 7 \\ 2 & -4.1 & -2 \\ 3 & 2 & 6 \end{bmatrix}$$

is a strictly diagonally dominant matrix as

Why? Because for each and every row, the answer to the question below is Yes.

Row 1: Is $\left|a_{11}\right| > \left|a_{12}\right| + \left|a_{13}\right|$? Yes, because

$\left|a_{11}\right| = |15|, \left|a_{12}\right| + \left|a_{13}\right| = |6| + |7| = 13, 15 > 13.$

Row 2: Is $\left|a_{22}\right| > \left|a_{21}\right| + \left|a_{23}\right|$? Yes, because

$\left|a_{22}\right| = |-4.1| = 4.1, \ \left|a_{21}\right| + \left|a_{23}\right| = |2| + |-2| = 4, \ 4.1 > 4$

Row 3: Is $\left|a_{33}\right| > \left|a_{31}\right| + \left|a_{32}\right|$? Yes, because

$\left|a_{33}\right| = |6|, \left|a_{31}\right| + \left|a_{32}\right| = |3| + |2| = 5, \ 6 > 5$

The matrix

$$[A] = \begin{bmatrix} 13 & 6 & 7 \\ 2 & -4.1 & -2 \\ 3 & 2 & 6 \end{bmatrix}$$

is a *not* a strictly diagonally dominant matrix as

Why? Because for each and every row, the answer to the question below is *not* a Yes.

Row 1: Is $|a_{11}| > |a_{12}| + |a_{13}|$? No, because

$|a_{11}| = |13|, |a_{12}| + |a_{13}| = |6| + |7| = 13, 13 \geq 13.$

Row 2: Is $|a_{22}| > |a_{21}| + |a_{23}|$? Yes, because

$|a_{22}| = |-4.1| = 4.1,\ \ |a_{21}| + |a_{23}| = |2| + |-2| = 4,\ 4.1 > 4$

Row 3: Is $|a_{33}| > |a_{31}| + |a_{32}|$? Yes, because

$|a_{33}| = |6|, |a_{31}| + |a_{32}| = |3| + |2| = 5,\ 6 > 5$

The matrix

$$[C] = \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}$$

is *not* a strictly diagonally dominant matrix.

Why? Because for each and every row, the answer to the question below is *not* a Yes.

Row 1: Is $|a_{11}| > |a_{12}| + |a_{13}|$? Yes, because

$|a_{11}| = |25|,\ \ |a_{12}| + |a_{13}| = |5| + |1| = 6,\ \ 25 > 6$

Row 2: Is $|a_{22}| > |a_{21}| + |a_{23}|$? No, because

$|a_{22}| = |8| = 8,\ \ |a_{21}| + |a_{23}| = |64| + |1| = 65,\ 8 < 65$

Row 3: Is $|a_{33}| > |a_{31}| + |a_{32}|$? No, because

$|a_{33}| = |1|, |a_{31}| + |a_{32}| = |144| + |12| = 156,\ \ 1 < 156$