Problem Statement

To make the fulcrum (Figure 1) of a bascule bridge, a long hollow steel shaft called the trunnion is shrink fit into a steel hub. The resulting steel trunnion-hub assembly is then shrink fit into the girder of the bridge.

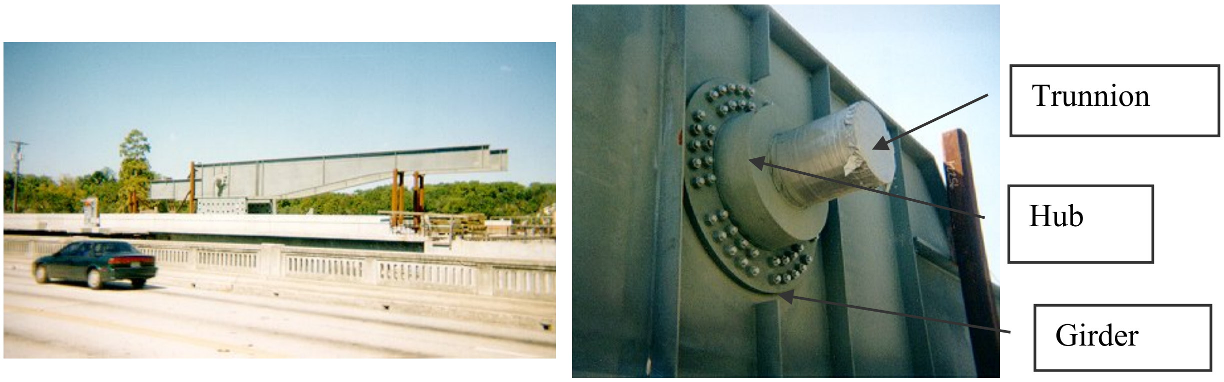

Figure 1 Trunnion-Hub-Girder (THG) assembly.

This is done by first immersing the trunnion in a cold medium such as dry-ice/alcohol mixture. After the trunnion reaches the steady state temperature of the cold medium, the trunnion outer diameter contracts. The trunnion is taken out of the medium and slid though the hole of the hub (Figure 2).

When the trunnion heats up, it expands and creates an interference fit with the hub. In 1995, on one of the bridges in Florida, this assembly procedure did not work as designed. Before the trunnion could be inserted fully into the hub, the trunnion got stuck. So a new trunnion and hub had to be ordered at a cost of $50,000. Coupled with construction delays, the total loss was more than hundred thousand dollars.

Why did the trunnion get stuck? This was because the trunnion had not contracted enough to slide through the hole.

Now the same designer is working on making the fulcrum for another bascule bridge. Can you help him/her so that he does not make the same mistake?

For this new bridge, he needs to fit a hollow trunnion of outside diameter $12.363''$ in a hub of inner diameter $12.358''$. His plan is to put the trunnion in dry ice/alcohol mixture (temperature of the fluid - dry ice/alcohol mixture is $-108°\text{F}$) to contract the trunnion so that it can be slided through the hole of the hub. To slide the trunnion without sticking, he has also specified a diametrical clearance of at least $0.01''$ between the trunnion and the hub. What temperature does he need to cool the trunnion to so that he gets the desired contraction?

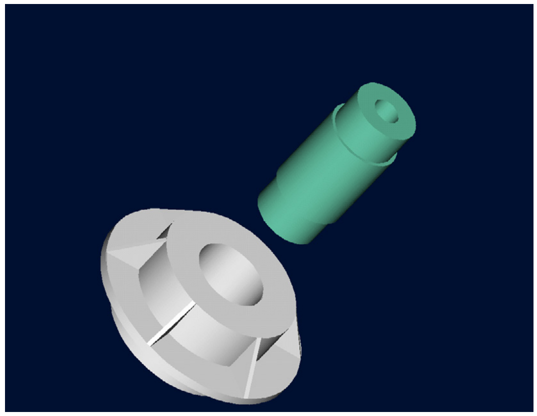

Figure 2 Trunnion slided through the hub after contracting

Solution

The reduction, ΔD in the outer diameter of the trunnion is given by

$$\Delta D = D \int_{T_{room}}^{T_{fluid}} \alpha dT \qquad (1)$$

where

D = outer diameter of the trunnion,
$\alpha =$ coefficient of thermal expansion coefficient at room temperature,
T_f = temperature of fluid needed,
T_{room} = room temperature.

Given

D = 12.363"
$\alpha =$ discrete data of coefficients of thermal expansion vs temperature is given in Figure 3
$T_{room} = 80°F$

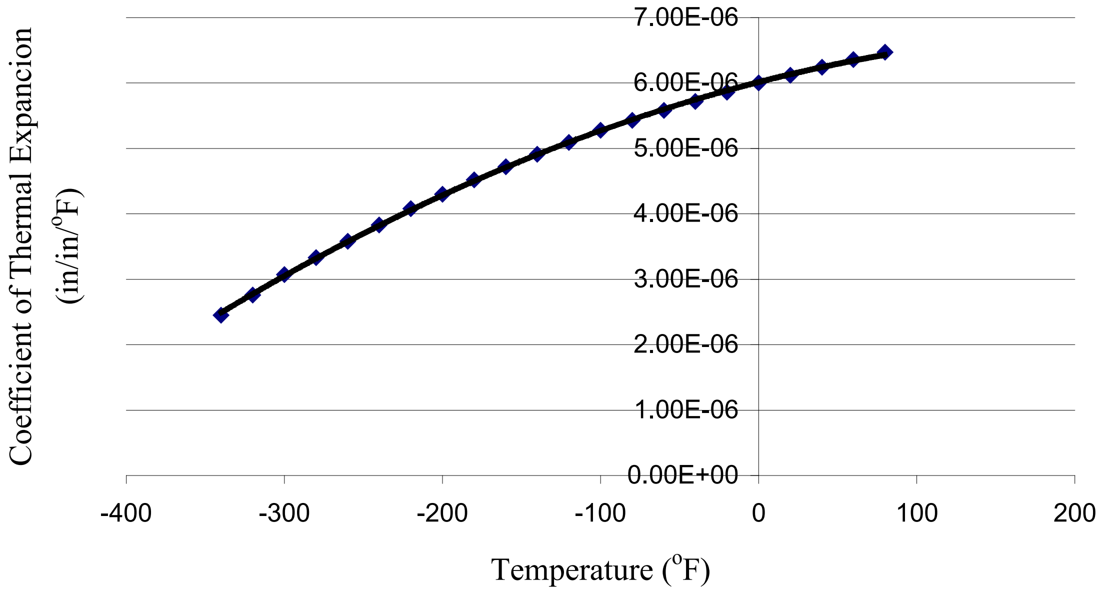

Figure 3 Varying thermal expansion coefficient as a function of temperature for cast steel.

In Figure 3, the thermal expansion coefficient of a typical cast steel is approximated by a second order polynomial[1] as

$$\alpha = -1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.0150 \times 10^{-6} \quad (2)$$

Since the desired contraction is at least $0.015''$, that is, $\Delta D = -0.015''$,

$$-0.015 = 12.363 \int_{80}^{T_f} \left(-1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.015 \times 10^{-6}\right) dT$$

$$-0.015 = 12.363 \left[-1.2278 \times 10^{-11} \frac{T^3}{3} + 6.1946 \times 10^{-9} \frac{T^2}{2} + 6.015 \times 10^{-6} T\right]_{80}^{T_f}$$

$$-0.015 = 12.363(-0.40927 \times 10^{-11} T_f^3 + 0.30973 \times 10^{-8} T_f^2 + 0.60150 \times 10^{-5} T_f$$
$$- 0.49893 \times 10^{-3})$$

$$\begin{aligned} \mathrm{f}(T_f) = {} & -0.50598 \times 10^{-10} T_f^3 + 0.38292 \times 10^{-7} T_f^2 + 0.74363 \times 10^{-4} T_f \\ & + 0.88318 \times 10^{-2} = 0 \end{aligned} \quad (3)$$

One can solve the nonlinear equation (3) to find the minimum fluid temperature needed to cool down the trunnion and get the desired contraction.

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[1] The second order polynomial is derived using regression analysis which is another mathematical procedure where numerical methods are employed. Regression analysis approximates discrete data such as the thermal expansion coefficient vs. temperature data as a continuous function. This is an excellent example of where one has to use numerical methods of more than one procedure to solve a real life problem.

The roots of Equation (3) are given as

$$-128.8°F, -802.9°F, 1688.5°F$$

Only one root is acceptable, that is,

$$T_f = -128.8°F$$

as $-802.9°F$ is less than absolute zero and $1688.5°F$ is above room temperature where the trunnion diameter would expand and not contract if heated. The contractor's plan to dip the trunnion in dry-ice/alcohol mixture temperature of $-108°$ F is not going to work.